1.1 Limits Graphically

What is a limit?

A limit is the _________ a function ____________ from both the left and the right side of a given ____________.

Example 1

Limit: (geeky math definition for Mr. Kelly)
Given a function $f$, the limit of $f(x)$ as $x$ approaches $c$ is a real number $R$ if $f(x)$ can be made arbitrarily close to $R$ by taking $x$ sufficiently close to $c$ (but not equal to $c$). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.

What is a one-sided limit?

A one-sided limit is the _________ a function approaches as you approach a given ____________ from either the _____ or _____ side.

Example 2

“The limit of $f$ as $x$ approaches 3 from the left side is $-1$."

$\lim_{x \to 3^-} f(x) = -1$

“The limit of $f$ as $x$ approaches 3 from the right side is 2."

$\lim_{x \to 3^+} f(x) = 2$
1.1 Limits Graphically

Example 3

a. \( \lim_{x \to 2^-} f(x) = \)  
   b. \( \lim_{x \to 2^+} f(x) = \)  
   c. \( \lim_{x \to 2} f(x) = \)

   d. \( \lim_{x \to 1} f(x) = \)  
   e. \( \lim_{x \to 0} f(x) = \)  
   f. \( \lim_{x \to 3} f(x) = \)

   g. \( \lim_{x \to -1} f(x) = \)  
   h. \( \lim_{x \to -3} f(x) = \)  
   i. \( f(-2) = \)

j. \( f(1) = \)

When does a limit not exist?

1.  
2.  
3.

Example 4

Sketch a graph of a function \( g \) that satisfies all of the following conditions.

   a. \( g(3) = -1 \)  
   b. \( \lim_{x \to 3} g(x) = 4 \)  
   c. \( \lim_{x \to 2^-} g(x) = 1 \)  
   d. \( g \) is increasing on \(-2 < x < 3\)  
   e. \( \lim_{x \to 2^-} g(x) > \lim_{x \to 2^+} g(x) \)

Example 5

Write T (true) or F (false) under each statement. Use the graph on the right.

<table>
<thead>
<tr>
<th>a. ( \lim_{x \to 1^-} f(x) = 1 )</th>
<th>b. ( \lim_{x \to 2} f(x) = 2 )</th>
<th>c. ( \lim_{x \to 1^+} f(x) = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>d. ( \lim_{x \to 1^-} f(x) = 2 )</td>
<td>e. ( \lim_{x \to 1^+} f(x) = ) does not exist</td>
<td></td>
</tr>
<tr>
<td>f. ( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) )</td>
<td>g. ( \lim_{x \to 2} f(x) = ) does not exist</td>
<td></td>
</tr>
</tbody>
</table>
1.1 Limits Graphically

For 1-5, give the value of each statement. If the value does not exist, write “does not exist” or “undefined.”

1.
   a. \( \lim_{x \to 1^-} f(x) = \)
   b. \( f(1) = \)
   c. \( \lim_{x \to 0} f(x) = \)

   d. \( \lim_{x \to 2^+} f(x) = \)
   e. \( f(-1) = \)
   f. \( f(2) = \)

   g. \( \lim_{x \to 1^+} f(x) = \)
   h. \( \lim_{x \to 1^-} f(x) = \)
   i. \( \lim_{x \to 2} f(x) = \)

2.
   a. \( \lim_{x \to 3^-} f(x) = \)
   b. \( f(1) = \)
   c. \( \lim_{x \to 1} f(x) = \)

   d. \( \lim_{x \to 2^-} f(x) = \)
   e. \( f(3) = \)
   f. \( \lim_{x \to 2} f(x) = \)

   g. \( \lim_{x \to 2} f(x) = \)
   h. \( f(-2) = \)
   i. \( f(4) = \)

3.
   a. \( \lim_{x \to 3} f(x) = \)
   b. \( f(3) = \)
   c. \( \lim_{x \to 0} f(x) = \)

   d. \( \lim_{x \to 3} f(x) = \)
   e. \( f(0) = \)
   f. \( \lim_{x \to 3} f(x) = \)

   g. \( \lim_{x \to 0} f(x) = \)
   h. \( f(1) = \)
   i. \( f(-1.6) = \)

4.
   a. \( \lim_{x \to 1^-} f(x) = \)
   b. \( f(2) = \)
   c. \( \lim_{x \to 2} f(x) = \)

   d. \( \lim_{x \to 1^-} f(x) = \)
   e. \( f(4) = \)
   f. \( \lim_{x \to 1} f(x) = \)

   g. \( \lim_{x \to 1^+} f(x) = \)
   h. \( f(1) = \)
   i. \( \lim_{x \to 4} f(x) = \)
5. a. \( \lim_{x \to 3} f(x) = \)  
   b. \( f(-1) = \)  
   c. \( \lim_{x \to 3} f(x) = \)  

   d. \( \lim_{x \to -1} f(x) = \)  
   e. \( f(-3) = \)  
   f. \( \lim_{x \to 3^+} f(x) = \)  

   g. \( f(3) = \)  
   h. \( \lim_{x \to 0} f(x) = \)  
   i. \( f(-4) = \)

6. Sketch a graph of a function \( f \) that satisfies all of the following conditions.
   a. \( f(-2) = 5 \)
   b. \( \lim_{x \to -2} f(x) = 1 \)
   c. \( \lim_{x \to 4^-} f(x) = 3 \)
   d. \( f \) is increasing on \( x < -2 \)
   e. \( \lim_{x \to 4^-} f(x) < \lim_{x \to 4^+} f(x) \)

7. Sketch a graph of a function \( g \) that satisfies all of the following conditions.
   a. \( g(1) = 3 \)
   b. \( \lim_{x \to -1} g(x) = -2 \)
   c. \( \lim_{x \to 3^-} g(x) = 5 \)
   d. \( g \) is increasing only on \(-5 < x < -3 \) and \( x > 1 \)
   e. \( \lim_{x \to 3^-} g(x) > \lim_{x \to 3^+} g(x) \)

8. Sketch a graph of a function \( h \) that satisfies all of the following conditions.
   a. \( \lim_{x \to 3} h(x) = h(-2) = 1 \)
   b. \( h(3) \) is undefined.
   c. \( \lim_{x \to -2^-} h(x) < \lim_{x \to -2^+} h(x) \)
   d. \( h \) is constant on \(-2 < x < 3 \) and decreasing everywhere else.
1. The graph of the function \( f \) is shown. Which of the following statements about \( f \) is true?

(A) \( \lim_{x \to a} f(x) = \lim_{x \to b} f(x) \)  
(B) \( \lim_{x \to a} f(x) = 4 \)  
(C) \( \lim_{x \to b} f(x) = 4 \)  
(D) \( \lim_{x \to b} f(x) = 1 \)  
(E) \( \lim_{x \to a} f(x) \) does not exist.

2. The figure below shows the graph of a function \( f \) with domain \( 0 \leq x \leq 4 \). Which of the following statements are true?

I. \( \lim_{x \to 2^-} f(x) \) exists.  
II. \( \lim_{x \to 2^+} f(x) \) exists.  
III. \( \lim_{x \to 2} f(x) \) exists.

(A) I only  
(B) II only  
(C) I and II only  
(D) I and III only  
(E) I, II, and III

3. If \([x]\) represents the greatest integer that is less than or equal to \( x \), then \( \lim_{x \to 0^-} \frac{2}{[x]} = \)

(A) \(-2\)  
(B) \(-1\)  
(C) 0  
(D) 2  
(E) the limit does not exist

4. Consider the function \( y = f(x) \) shown below. Which of the following statements is true?

(A) \( \lim_{x \to 1} f(x) = 3 \)  
(B) \( f(1) = 1 \)  
(C) \( f(x) \) is continuous for all \( x \).  
(D) \( \lim_{x \to 1} f(x) = f(1) \)  
(E) None of the above