PRACTICE

The boundaries on problems #3, 8, and 9 are not good. The integral tries to integrate where the function is not defined. This is not possible. Huge bummer. I will be more careful on the quiz. Make sure you can still set up these integrals and change the boundaries to u.

Also, some of the integrals get pretty "fractiony" when you evaluate them. If you need a calculator to help simplify that is coolio. Other than that, this assignment was money.

(that's right an integral money to represent how money this assignment is)

Evaluate the definite integral.

1. $\int_0^1 \frac{x}{(x^2+1)^3}\,dx$

$u = x^2+1$

$\frac{du}{dx} = \frac{2x\,dx}{2x}$

$\frac{du}{2x} = dx$

$\int \frac{x}{u}\,dx \to \int \frac{\cancel{x}}{u^3}\frac{du}{2\cancel{x}} \to \frac{1}{2}\int_1^2 u^{-3}\,du$

$\frac{1}{2}\left[-\frac{1}{2}u^{-2}\right]_1^2$

$\frac{1}{2}\left[\left(-\frac{1}{2}\cdot\frac{1}{(2)^2}\right) - \left(-\frac{1}{2}\cdot\frac{1}{(1)^2}\right)\right]$

$\frac{1}{2}\left[\left(-\frac{1}{8}\right) + \left(+\frac{1}{2}\right)\right]$

$\frac{1}{2}\left(\frac{3}{8}\right) = \frac{3}{16}$

2. $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int \sin(u)\frac{du}{2} \to \frac{1}{2}\int_0^{\pi} \sin(u)\,du$

$\frac{1}{2}\left[-\cos(u)\right]_0^{\pi}$

$\frac{1}{2}\left(-\cos(\pi) - -\cos(0)\right)$

$\frac{1}{2}\left(-1 + 1\right)$

$\frac{1}{2}(2) = 1$

3. $\int_0^3 2x\sqrt{3x-5}\,dx$

$u = 3x-5$

$\frac{du}{3} = \frac{3\,dx}{3}$

$\frac{du}{3} = dx$

$u = 3x - 5$
$+5 \quad +5$

$\frac{u+5}{3} = \frac{3x}{3}$

$\frac{u+5}{3} = x$

$\int 2x\sqrt{u}\,dx \to \int 2x\sqrt{u}\,\frac{du}{3} \to \frac{2}{3}\int_{-5}^{4}\left(\frac{u+5}{3}\right)\sqrt{u}\,du$

$\frac{2}{9}\int_{-5}^{4}(u+5)\,u^{1/2}\,du$

$\frac{2}{9}\int_{-5}^{4} u^{3/2} + 5u^{1/2}\,du$

$\frac{2}{9}\left[\frac{2}{5}u^{5/2} + \frac{10}{3}u^{3/2}\right]_{-5}^{4}$

$\frac{2}{9}\left[\left(\frac{2}{5}\sqrt{4^5} + \frac{10}{3}\sqrt{4^3}\right) - \left(\frac{2}{5}\sqrt{-5^5} + \frac{10}{3}\sqrt{-5^3}\right)\right]$

impossible
Can't integrate!
function does not exist $x \le \frac{5}{3}$

4. $\int_1^{10} \sqrt{5x-1}\,dx$

$u = 5x-1$

$\frac{du}{5} = \frac{5\,dx}{5}$

$\frac{du}{5} = dx$

$\int \sqrt{u}\,\frac{du}{5} \to \frac{1}{5}\int_4^{49} u^{1/2}\,du$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^{49}$

$\frac{1}{2}\left[\frac{2}{3}\sqrt{49^3} - \frac{2}{3}\sqrt{4^3}\right]$

$\frac{1}{2}\left(\frac{2}{3}(7^3) - \frac{2}{3}(8)\right)$

$\frac{1}{2}\left(\frac{686}{3} - \frac{16}{3}\right) = \frac{1}{2}\left(\frac{620}{3}\right) = \frac{335}{3}$

Evaluate the definite integral.

5. $\int_{-2}^{3} \frac{1}{1+9t^2} dt$

INVERSE TRIG!

$u = 3t$

$\frac{du}{3} = \frac{3\,dt}{3}$

$\frac{du}{3} = dt$

$$\int \frac{1}{1+u^2} \frac{du}{3} \to \frac{1}{3} \int_{-6}^{9} \frac{1}{1+u^2} du$$

$$\frac{1}{3} \left[\tan^{-1}(u) \right]_{-6}^{9}$$

$$\frac{1}{3} \left(\tan^{-1}(9) - \tan^{-1}(6) \right)$$

6. $\int_{1}^{5} x\sqrt{2x-1}\,dx$

$u = 2x-1$

$\frac{du}{2} = \frac{\cancel{2}\,dx}{\cancel{2}}$

$\frac{du}{2} = dx$

$u = 2x - 1$
$+1 \quad\quad +1$

$\frac{u+1}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$\frac{u+1}{2} = x$

$$\int x \sqrt{u} \frac{du}{2} \to \frac{1}{2} \int_{1}^{9} \left(\frac{u+1}{2}\right) \sqrt{u}\, du \to \frac{1}{4} \int_{1}^{9} (u+1) u^{1/2} du$$

$$\frac{1}{4} \int_{1}^{9} u^{3/2} + u^{1/2} du$$

$$\frac{1}{4} \left[\frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]_{1}^{9}$$

$$\frac{1}{4} \left[\left(\frac{2}{5}\sqrt{9^5} + \frac{2}{3}\sqrt{9^3} \right) - \left(\frac{2}{5}\sqrt{1^5} + \frac{2}{3}\sqrt{1^3} \right) \right]$$

$$\frac{1}{4} \left[\left(\frac{2}{5}(3^5) + \frac{2}{3}(3^3) \right) - \left(\frac{2}{5} + \frac{2}{3} \right) \right]$$

$$\frac{1}{4} \left[\left(\frac{486}{5} + 18 \right) - \left(\frac{6}{15} + \frac{10}{15} \right) \right]$$

$$\frac{1}{4} \left[\frac{576}{5} - \frac{16}{15} \right]$$

$$\frac{1}{4} \left[\frac{1728}{15} - \frac{16}{15} \right]$$

$$\frac{1}{4} \left(\frac{1712}{15} \right)$$

$$\frac{1712}{60} = \frac{428}{15}$$

7. $\int_{3}^{6} (x^2 - 2x) dx$

don't need u sub!

$$\left. \frac{1}{3}x^3 - x^2 \right]_{3}^{6}$$

$$\left(\frac{1}{3}(6)^3 - 6^2 \right) - \left(\frac{1}{3}(3)^3 - 3^2 \right)$$

$$\left(\frac{216}{3} - 36 \right) - (9 - 9)$$

$$(72 - 36) - 0$$

$$36$$

8. $\int_{1}^{3} \frac{8x}{\sqrt{1-16x^2}} dx$

$u = 1 - 16x^2$

$\frac{du}{-32x} = \frac{-\cancel{32x}\,dx}{-\cancel{32x}}$

$\frac{du}{-32x} = dx$

$$\int \frac{8\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-\frac{32}{4}\cancel{x}} \to -\frac{1}{4} \int_{-15}^{-143} u^{-1/2} du$$

Oops! The boundaries are terrible!

The domain is $1 - 16x^2 > 0$

$$-\frac{1}{4} < x < \frac{1}{4}$$

Can only integrate where function is defined

My B!

Evaluate the definite integral.

9. $$\int_{\sqrt{\frac{\pi}{8}}}^{\sqrt{\frac{\pi}{4}}} \frac{-x\csc^2(2x^2)}{\cot(2x^2)}\,dx$$

$$\int \frac{-\cancel{x}\,\cancel{\csc^2(2x^2)}}{u} \cdot \frac{du}{-\cancel{\csc^2(2x^2)} \cdot 4\cancel{x}} \rightarrow -\frac{1}{4}\int_1^0 u^{-1}\,du \rightarrow \frac{1}{4}\int_0^1 u^{-1}\,du$$

$u = \cot(2x^2)$

$du = -\csc^2(2x^2) \cdot 4x\,dx$

$\frac{du}{-\csc^2(2x^2) \cdot 4x} = dx$

$\cot\left(2\left(\sqrt{\frac{\pi}{4}}\right)^2\right)$

$\cot\left(2\left(\frac{\pi}{4}\right)\right)$

$\cot\left(\frac{\pi}{2}\right)$

0

$\cot\left(2\left(\sqrt{\frac{\pi}{8}}\right)^2\right)$

$\cot\left(2\left(\frac{\pi}{8}\right)\right)$

$\cot\left(\frac{\pi}{4}\right)$

1

$\frac{1}{4}\left[\ln|u|\right]_0^1$

$\frac{1}{4}\left(\ln|1| - \ln|0|\right)$

$\frac{1}{4}(0 -$

DNE

Again, my B!

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10. $$\int_0^1 \frac{y^2 + 2y}{\sqrt[3]{y^3 + 3y^2 + 4}}\,dy$$

$$\int \frac{\cancel{y^2+2y}}{u^{1/3}} \cdot \frac{du}{\substack{(3y^2+6y) \\ 3\cancel{(y^2+2y)}}} \rightarrow \frac{1}{3}\int_4^8 u^{-1/3}\,du$$

$u = y^3 + 3y^2 + 4$

$\frac{du}{(3y^2+6y)} = \frac{(3y^2+6y)\,dy}{(3y^2+6y)}$

$\frac{du}{(3y^2+6y)} = dy$

$\frac{1}{3}\left[\frac{3}{2}u^{2/3}\right]_4^8$

$\frac{1}{3}\left(\frac{2}{3}\sqrt[3]{8^2} - \frac{2}{3}\sqrt[3]{4^2}\right)$

$\frac{1}{3}\left(\frac{2}{3}(4) - \frac{2}{3}\sqrt[3]{16}\right)$

$\frac{1}{3}\left(\frac{8}{3} - \frac{2}{3}\sqrt[3]{16}\right)$

$\frac{8}{9} - \frac{2}{9}\sqrt[3]{16}$

Yeah, it worked ☺

10.3 u Substitution Definite Integrals

TEST PREP

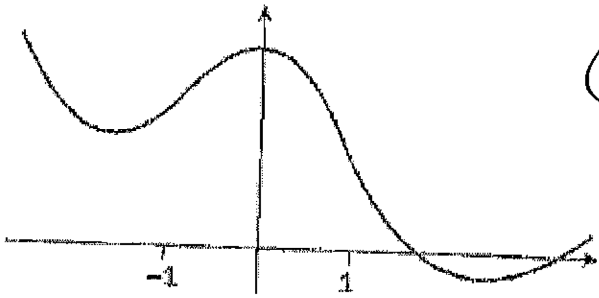
MULTIPLE CHOICE

5. The cost of producing x units of a certain item is $c(x) = 2000 + 8.6x + 0.5x^2$. What is the instantaneous rate of change of c with respect to x when $x = 300$?

(A) 313.6
(B) 308.6
(C) 300.0
(D) 297.2
(E) 200.0

B

12. The graph of $y = f(x)$ is given below.



C

Which of the following is true?

(A) The graph is concave down (for all values of x).

(B) The graph is concave up (for all values of x).

(C) The graph is concave up for $x > 1$ and concave down for $-1 < x < 1$.

(D) The graph is concave up for $-1 < x < 1$ and concave down for $x < -1$.

(E) Nothing can be said about the concavity of the graph above without knowing the rule for the function.

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9. Evaluate $\int_{-1}^{2} (3x^2 - 4x + 2)\,dx$.

C

(A) -2

(B) 14

(C) 9

(D) 18

(E) 21

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13. Let $f(x) = \pi \sec^2 \pi x - 1$. Which of the following statements is true?

(A) An antiderivative of f is $F(x) = \tan \pi x$ and $\int_1^3 f(x)\,dx = -2$.

(B) An antiderivative of f is $F(x) = \tan \pi x$ and $\int_1^3 f(x)\,dx$ is undefined.

(C) An antiderivative of f is $F(x) = \tan \pi x + 3$ and $\int_1^3 f(x)\,dx = -2$.

(D) An antiderivative of f is $F(x) = \tan \pi x - x$ and $\int_1^3 f(x)\,dx = -2$.

(E) An antiderivative of f is $F(x) = \tan \pi x - x + 3$ and $\int_1^3 f(x)\,dx$ is undefined.

E

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14. An equivalent representation of the definite integral $\int_1^3 2\cos(x^2)dx$ is

B

(A) $\int_1^3 \cos u \; du$

(B) $\int_1^9 \cos u \; du$

(C) $\int_1^{\sqrt{3}} \cos u \; du$

(D) $\int_1^9 2\sqrt{u} \cos u \; du$

(A) $\int_1^{\sqrt{3}} 2\sqrt{u} \cos u \; du$