Name: Solutions Date:__________ Period: _____ **Review**

Unit 11 Review – Area and Volume

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 11.

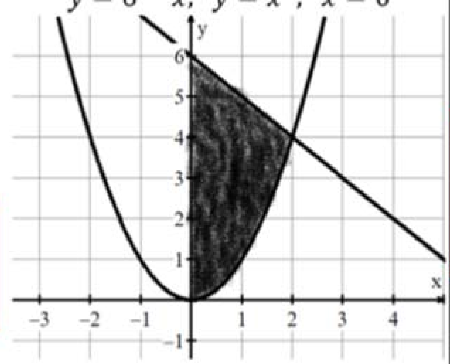
11.1 Area Between Two Curves

$$A = \int_a^b [f(x) - g(x)]\,dx$$

$f(x) \geq g(x)$ for all x in $[a, b]$

Set up the integral to find the area of the shaded region, but DO NOT EVALUATE.

$y = 6 - x,\ y = x^2,\ x = 0$



1. With respect to x.

$\int_0^2 (6-x) - x^2\,dx$

$\boxed{\int_0^2 (-x^2 - x + 6)\,dx}$

2. With respect to y.

$\boxed{\int_0^4 \sqrt{y}\,dy + \int_4^6 6-y\,dy}$

11.2 Volume – Disc Method

$$V = \pi \int_a^b [R(x)]^2\,dx$$

where $R(x)$ is the "distance" between the axis of revolution and the outside of the object.

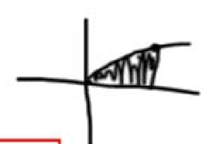
Set up the integral to find the volume of the bounded area that revolves about the given line. DO NOT EVALUATE.

3. $y = 4 - x^2,\ x \geq -1,\ y = 0$ about the x-axis.

$\boxed{V = \pi \int_{-1}^{2} (4 - x^2)^2\,dx}$

4. $y = \sqrt{x},\ y = 0,\ x = 4$ about the line $x = 4$.

$y^2 = x$



$\boxed{V = \pi \int_0^2 (y^2 - 4)^2\,dy}$

11.3 Volume – Washer Method

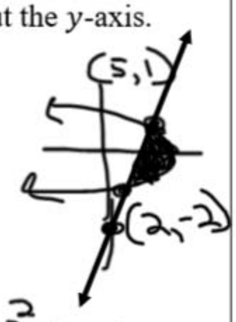
$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx$$

where $R(x)$ is the radius of the **OUTSIDE** of the object and
$r(x)$ is the radius of the **INSIDE** of the object.

Find the volume of the bounded region when rotating it around the given axis. Round answers to three decimal places.

5. $x = 6 - y^2$ and $y = x - 4$ about the y-axis.

$y+4=x$



$V=\pi\int_{-2}^{1}(6-y^2)^2-(y+4)^2\,dy \approx$

$\boxed{124.407}$

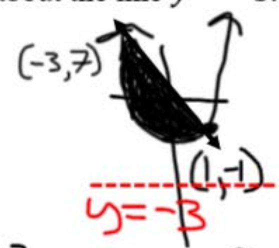
6. $y = x^2 - 2$, $y = -2x + 1$ about the line $y = -3$.

$x^2-2=-2x+1$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$



$V=\pi\int_{-3}^{1}(-2x+1+3)^2+(x^2-2+3)^2$

$\boxed{V\approx 294.891}$

11.4 Perpendicular Cross Sections

$$V = \int_a^b A(x) \, dx$$

where $\boldsymbol{A(x)}$ is the **AREA** of a cross section perpendicular to the x-axis.

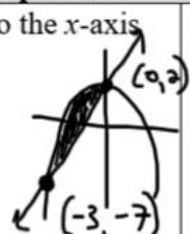
Use the area bounded by $y = 2 - x^2$ and $y = 3x + 2$ as the base of a solid with the indicated cross sections. Round answers to 3 decimal places.

$\frac{\pi r^2}{2}$

7. Equilateral triangles perpendicular to the x-axis

$A = \frac{\sqrt{3}}{4}s^2$

$\int_{-3}^{0}\frac{\sqrt{3}}{4}\left[(2-x^2)-(3x+2)\right]^2dx$



$\boxed{V\approx 3.507}$

8. Semi-circles perpendicular to the y-axis. y-axis!!

$x=\pm\sqrt{2-y} \qquad x=\frac{y-2}{3}$

$\frac{\pi}{2}\int_{-7}^{2}r^2=\frac{\pi}{2}\int_{-7}^{2}\left[\frac{\left(\frac{y-2}{3}\right)-\left(-\sqrt{2-y}\right)}{2}\right]^2dy$

$\boxed{V\approx 1.060}$

Test Prep: 9-D, 10-E, 11-D, 12-E, 13-B, 14-D, 15-B, 16-B.