2.2 Definition of the Derivative

**Recall:** Average rate of change =

Average rate of change on the interval [ ] is ___________

**Definition of the Derivative:**
This limit gives an expression that calculates the instantaneous rate of change (slope of the tangent line) of $f(x)$ at any given $x$-value.

$$f'(x) =$$

**Notation for the Derivative:**

<table>
<thead>
<tr>
<th>Lagrange</th>
<th>Leibniz</th>
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**Find the derivative using the Definition of the Derivative (limits).**

1. $f(x) = 2x^2 - 7x + 1$
2. $y = \frac{1}{x}$
3. If $f$ represents how many meters you have run and $x$ represents the minutes, describe in full sentences the following:

$$f(8) = 1,500 \quad \quad \quad f'(3) = 161$$
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Alternate Definition – Derivative at a Point:
Finding the derivative at a specific $x$-value ($x = c$).

\[ f'(c) = \text{ or } f'(c) = \]

4. Find $f'(-2)$ if $f(x) = 2x^2 + 1$.

5. $f(x) = x^3 - \frac{3}{x}$ and $f'(x) = 3x^2 + \frac{3}{x^2}$
   Find the equation of the tangent line at $x = 2$.

Identify the original function $f(x)$, and what value of $c$ to evaluate $f'(c)$.

6. \[ \lim_{h \to 0} \frac{3 \ln(2+h) - 3 \ln 2}{h} \]

7. \[ \lim_{x \to 7} \frac{1}{\sqrt{x^2 - 2x}} - \frac{1}{\sqrt{35}} \]
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Find the derivative using limits. If the equation is given as \( y = \), use Leibniz Notation: \( \frac{dy}{dx} \). If the equation is given as \( f(x) = \), use Lagrange Notation: \( f'(x) \). WRITE SMALL!!

1. \( f(x) = 7 - 6x \)  
2. \( y = 5x^2 - x \)  
3. \( y = x^2 + 2x - 9 \)

4. \( y = \sqrt{5x} + 2 \)

5. \( f(x) = \frac{1}{x-2} \)

For each problem, create an equation of the tangent line of \( f \) at the given point. Leave in point-slope.

6. \( f(7) = 5 \) and \( f'(7) = -2 \)
7. \( f(-2) = 3 \) and \( f'(-2) = 4 \)
8. \( f(x) = 3x^2 + 2x; \) \( f'(x) = 6x + 2; \ x = -2 \)
9. \( f(x) = 10\sqrt{6x+1}; \quad f'(x) = \frac{30}{\sqrt{6x+1}}; \quad x = 4 \)

10. \( f(x) = \cos{2x}; \quad f'(x) = -2\sin{2x}; \quad x = \frac{\pi}{4} \)

11. \( f(x) = \tan{x}; \quad f'(x) = \sec^2{x}; \quad x = \frac{\pi}{3} \)

12. \( \lim_{h \to 0} \frac{3(1+h)^2 - 7(1+h) + 1 + 3}{h} \)

13. \( \lim_{h \to 0} \frac{\log(2-4(h-5)) - \log(22)}{h} \)

14. \( \lim_{x \to -2} \frac{(3x-9x^2) + 42}{x+2} \)

15. \( \lim_{x \to 5} \frac{1}{\sqrt{3x}} - \frac{1}{\sqrt{15}} \)

16. \( \lim_{h \to 0} \frac{e^{6(3+h)+1} - e^{19}}{h} \)

17. \( \lim_{x \to \frac{\pi}{2}} \frac{6x^2 \sin{x} - \frac{3}{2}\pi^2}{x - \frac{\pi}{2}} \)

**Identify the original function \( f(x) \), and what value of \( c \) to evaluate \( f'(c) \).**

18. \( C \) is the number of championships Sully has won while coaching basketball. 
   \( t \) is the number of years since 2002 for the function \( C(t) \). 
   \( C(12) = 3 \) and \( C'(12) = 0.4 \)

19. \( d \) is the distance (in miles) from home when you walk to school. 
   \( h \) is the number of hours since 7:00 a.m. for the function \( d(h) \). 
   \( d(0.2) = 0.5 \) and \( d'(0.2) = -11 \)
20. \( W \) is the number of cartoon shows Mr. Kelly watches every week. 
\( x \) is the number of children Mr. Kelly has for the function \( W(x) \).
\( W(7) = 25 \) and \( W'(7) = 3 \)

21. \( g \) is the number of gray hairs on Mr. Brust’s head. 
\( x \) is the number of students in his 4th period.
\( g(26) = 501 \) and \( g'(15) = 130 \)

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1. Let \( f'(x) = \lim_{h \to 0} \frac{(x+h)^2-x^2}{h} \). For what value of \( x \) does \( f(x) = 4 \)?

(A) \(-4\)  (B) \(-1\)  (C) \(1\)  (D) \(2\)  (E) \(4\)

2. If \( f(x + y) = f(x) \cdot f(y) \) and if \( \lim_{h \to 0} \frac{f(h)-1}{h} = 6 \), then \( f'(x) = \)

(A) \(6\)  (B) \(6 + f(x)\)  (C) \(6 \cdot f(x)\)
(D) \(6 + f(h)\)  (E) \(6 \cdot f(h)\)

3. Which of the following gives the derivative of the function \( f(x) = x^2 \) at the point \((2, 4)\)?

(A) \( \lim_{h \to 0} \frac{(x+2)^2-x^2}{4} \)  (B) \( \lim_{h \to \infty} \frac{(2+h)^2-2^2}{h} \)  (C) \( \frac{(2+h)^2-2^2}{h} \)
(D) \( \lim_{h \to 0} \frac{(2+h)^2-2^2}{h} \)  (E) \( \lim_{h \to 0} \frac{(4+h)^2-4^2}{h} \)