3.2 Product and Quotient Rule

Find the derivative.
\[ f(x) = (x + 4)(2x - 5) \]

**PRODUCT RULE**
\[
\frac{d}{dx} (uv) = \]

Find the derivative of the following.
\[ f(x) = (3x^2 + 2x - 3)(x - 1) \]
\[ y = (2x^{-3} + 4x + \pi)(4x + 1) \]

Evaluate
\[ f(x) = \sqrt{x}(3x^2 - 3) \]
Find \( f'(4) \)

Find the derivative.
\[ f(x) = \frac{x - 5}{2x + 1} \]

**QUOTIENT RULE**
\[
\frac{d}{dx} (uv) = \]
Find the derivative of the following.

\[ f(x) = \frac{3x + 1}{2x^2} \]
\[ y = \frac{2x^2}{3x + 1} \]

Horizontal Tangents

Find all horizontal tangents for \( y = \frac{2x^2}{3x + 1} \)

Find \( f'(4) \) given the following:

\( g(4) = 3 \) and \( g'(4) = -2 \)
\( h(4) = -1 \) and \( h'(4) = 5 \)

\[ f(x) = g(x) - h(x) \quad f(x) = h(x) + 2 \quad f(x) = g(x) + 2h(x) \]

\[ f(x) = \frac{h(x)}{g(x)} \quad f(x) = g(x)h(x) \]

**SUMMARY:**

Now, summarize your notes here!
### 3.2 Product and Quotient Rule

<table>
<thead>
<tr>
<th>Find the derivative of the following.</th>
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<tbody>
<tr>
<td>1. ( f(x) = \frac{5x-2}{x^2+1} )</td>
<td>7. ( y = \frac{x}{x-1} )</td>
</tr>
<tr>
<td>2. ( g(x) = (2x + 1)(x^3 - 1) )</td>
<td>8. ( y = x^{-2}(ex^3 + 3) )</td>
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<tr>
<td>3. ( y = (3x^2 - 2x)(x^2 + 3x - 4) )</td>
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<tr>
<td>4. ( h(x) = \frac{6x^2+3x-5}{3x} )</td>
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<tr>
<td>5. ( f(t) = \frac{t+1}{\sqrt{t}} )</td>
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<tr>
<td>6. ( f(r) = r^2(5r^3 + 3) )</td>
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</table>

\[
\frac{dy}{dx} =
\]

\[
\frac{d^2y}{dx^2} =
\]

\[
y' =
\]

\[
y'' =
\]
Given \( f(x) = (x^2 - 5)(3x + 2) \), find the following.

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<tbody>
<tr>
<td>9.</td>
<td>( f'(2) = )</td>
</tr>
<tr>
<td>10.</td>
<td>Find the slope of ( f(x) ) at ( x = -3 ).</td>
</tr>
<tr>
<td>11.</td>
<td>What is the slope of the tangent line of ( f(x) ) at the point ( (4, 48) )?</td>
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</table>

Is the slope of the tangent line positive, negative, or zero at the given point?

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<td>12.</td>
<td>( f(x) = \frac{2 - x}{x - 3} ) at ( x = 4 )</td>
</tr>
<tr>
<td>13.</td>
<td>( g(x) = (x + 1)^2 ) at ( x = -4 )</td>
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</table>

Determine the \( x \)-values (if any) at which the function has a horizontal tangent line.

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<td>14.</td>
<td>( f(x) = \frac{4x^2 - 10x^2}{2x} )</td>
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<tr>
<td>15.</td>
<td>( g(x) = \frac{x^2}{x + 1} )</td>
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Write the equation of the tangent line and the normal line at the point given.

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<td>16.</td>
<td>( f(x) = \frac{x - 1}{x + 1} ) at ( x = 2 )</td>
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</table>

Find \( f'(2) \) given the following.

\[
g(2) = 3 \quad \text{and} \quad g'(2) = -2 \\
h(2) = -1 \quad \text{and} \quad h'(2) = 4
\]

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<td>17.</td>
<td>( f(x) = 2g(x) + h(x) )</td>
</tr>
<tr>
<td>18.</td>
<td>( f(x) = 4 - h(x) )</td>
</tr>
<tr>
<td>19.</td>
<td>( f(x) = \frac{g(x)}{h(x)} )</td>
</tr>
<tr>
<td>20.</td>
<td>( f(x) = g(x)h(x) )</td>
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</table>
MULTIPLE CHOICE

1. Suppose \( f(x) \) is a differentiable function with \( f(1) = 2, f(2) = -2, f'(2) = 5, f'(1) = 3, \) and \( f(5) = 1. \)
   An equation of a line tangent to the graph of \( f \) is
   
   (A) \( y - 3 = 2(x - 1) \)
   (B) \( y - 2 = (x - 1) \)
   (C) \( y - 3 = 5(x - 1) \)
   (D) \( y - 2 = 3(x - 1) \)
   (E) \( y - 1 = 5(x - 2) \)

2. Let \( f \) and \( g \) be differentiable functions with the following properties:
   I. \( f(x) < 0 \) for all \( x \)
   II. \( g(5) = 2 \)
    If \( h(x) = \frac{f(x)}{g(x)} \) and \( h'(x) = \frac{f'(x)}{g(x)} \), then \( g(x) = \)
    
    (A) \( \frac{1}{f'(x)} \)
    (B) \( f(x) \)
    (C) \( -f(x) \)
    (D) \( 0 \)
    (E) \( 2 \)

3. At what point on the graph of \( y = \frac{1}{2}x^2 - \frac{3}{2} \) is the tangent line parallel to the line \( 4x - 8y = 5 \) ?
   
   (A) \( \left( \frac{1}{2}, -\frac{3}{8} \right) \)
   (B) \( \left( \frac{1}{2}, -\frac{11}{8} \right) \)
   (C) \( \left( 2, \frac{3}{8} \right) \)
   (D) \( \left( 2, \frac{1}{2} \right) \)
   (E) \( \left( -\frac{1}{2}, -\frac{11}{8} \right) \)

4. If \( f(x) \) is continuous and differentiable and \( f(x) = \begin{cases} ax^4 + 5x; & x \leq 2 \\ bx^2 - 3x; & x > 2 \end{cases} \), then find the value of \( b. \)
   
   (A) 0.5
   (B) 0
   (C) 2
   (D) 6
   (E) There is no value of \( b. \)
5. Which of the following functions are continuous but not differentiable at \( x = 0 \)?
   
   I. \( f(x) = x^{\frac{1}{3}} \)
   II. \( g(x) = |x| \)
   III. \( h(x) = x|x| \)

   (A) I only
   (B) II only
   (C) I and II
   (D) II and III
   (E) I, II, and III

**FREE RESPONSE**

1. A continuous function \( g \) is defined on the closed interval \(-8 \leq x \leq 6\) and is shown above.

   (a) Find the approximate value of \( g'(4) \). Show the computations that lead to your answer.

   (b) Let \( h \) be the function defined by \( h(x) = \frac{g(x)}{x^2 + 1} \). Find \( h'(-2) \).