3.2 Product and Quotient Rule

PRACTICE

Find the derivative of the following.

1. $f(x) = \frac{5x-2}{x^2+1} = \frac{u}{v}$ $\quad f'(x) = \frac{u'v - uv'}{v^2}$

$$\frac{5(x^2+1) - (5x-2)(2x)}{(x^2+1)^2}$$

$$\frac{5x^2+5-10x^2+4x}{(x^2+1)^2}$$

$$f'(x) = \frac{-5x^2+4x+5}{(x^2+1)^2}$$

2. $g(x) = (2x+1)(x^3-1)$

$$g'(x) = 8x^3+3x^2-2$$

3. $y = (\overset{u}{3x^2-2x})(\overset{v}{x^2+3x-4})$ $\quad y' = u'v + uv'$

$$y' = (6x-2)(x^2+3x-4) + (3x^2-2x)(2x+3)$$

this is simplified enough
you could multiply it out if you really wanted to!

$$y' = 12x^3+21x^2-36x+8$$

4. $h(x) = \frac{6x^2+3x-5}{3x}$

$$h'(x) = 2 + \frac{5}{3x^2}$$

5. $f(t) = \frac{t+1}{\sqrt{t}} = \frac{t}{t^{1/2}} + \frac{1}{t^{1/2}}$

$$f(t) = t^{1/2} + t^{-1/2}$$

$$f'(t) = \frac{1}{2}t^{-1/2} - \frac{1}{2}t^{-3/2}$$

$$f'(t) = \frac{1}{2\sqrt{t}} - \frac{1}{2\sqrt{t^3}}$$

6. $f(r) = r^2(5r^3+3)$

$$f'(r) = 25r^4+6r$$

Find the derivatives of the following.

7. $y = \frac{x}{x-1} = \frac{u}{v}$ $\quad y' = \frac{u'v - uv'}{v^2}$

$$\frac{dy}{dx} = \frac{(1)(x-1) - x(1)}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{(x-1)^2} = \frac{-1}{(x-1)(x-1)} = \frac{-1}{x^2-2x+1} = \frac{u}{v}$$

$$\frac{0(x^2-2x+1) - (-1)(2x-2)}{(x^2-2x+1)^2}$$

$$\frac{2x-2}{(x^2-2x+1)^2}$$

8. $y = x^{-2}(ex^3+3)$

$$y' = e - \frac{6}{x^3}$$

$$y'' = \frac{18}{x^4}$$

Given $f(x) = (x^2 - 5)(3x + 2)$, find the following.

9. $f'(2) =$ $u'v + uv'$

$(2x)(3x+2) + (x^2-5)(3)$

$6x^2 + 4x + 3x^2 - 15$

$f'(x) = 9x^2 + 4x - 15$

$9(2)^2 + 4(2) - 15$

$36 + 8 - 15$

$f'(2) = 29$

10. Find the slope of $f(x)$ at $x = -3$.

$f'(-3) = 54$

11. What is the slope of the tangent line of $f(x)$ at the point (4, 48) ?

$f'(x) = 9x^2 + 4x - 15$

$f'(4) = 9(4)^2 + 4(4) - 15$

$144 + 16 - 15$

$f'(4) = 145$

Is the slope of the tangent line positive, negative, or zero at the given point?

12. $f(x) = \frac{2-\frac{1}{x}}{x-3}$ at $x = 4$

$f'(4) = \frac{-27}{16}$ Negative

13. $g(x) = (x+1)^2$ at $x = -4$

$g(x) = (x+1)(x+1)$

$g(x) = x^2 + 2x + 1$

$g'(x) = 2x + 2$

$g'(-4) = 2(-4) + 2$

$g'(-4) = -6$ Negative!

Determine the x-values (if any) at which the function has a horizontal tangent line.

14. $f(x) = \frac{4x^3 - 10x^2}{2x}$

$x = \frac{5}{4}$

15. $g(x) = \frac{x^2}{x+1}$ $\frac{u}{v}$ $g'(x) = 0$

$g'(x) = \frac{u'v - uv'}{v^2}$

$g'(x) = \frac{(2x)(x+1) - (x^2)(1)}{(x+1)^2}$

$0 = \frac{2x^2 + 2x - x^2}{(x+1)^2}$

When you set a fraction equal to zero, the denominator doesn't matter.
If the numerator equals zero then it doesn't matter what you are dividing by!!

$0 = x^2 + 2x$

$0 = x(x+2)$

$x = 0, -2$

Write the equation of the tangent line and the normal line at the point given.

16. $f(x) = \frac{x-1}{x+1}$ at $x = 2$

Tangent Line

$y - \frac{1}{3} = \frac{2}{9}(x-2)$

Normal Line

$y - \frac{1}{3} = -\frac{9}{2}(x-2)$

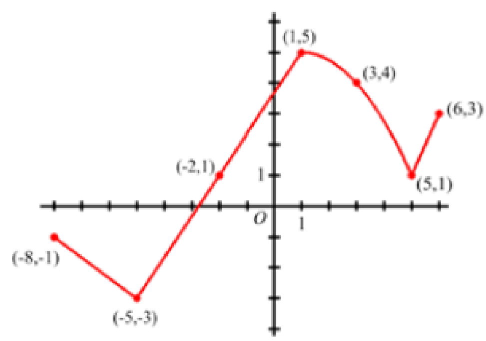
Find $f'(2)$ given the following.

$g(2) = 3$ and $g'(2) = -2$ $h(2) = -1$ and $h'(2) = 4$	17. $f(x) = 2g(x) + h(x)$ $f' = 2g' + h'$ $f'(2) = 2(-2) + 4$ $f'(2) = 0$	18. $f(x) = 4 - h(x)$ $f'(2) = -4$
	19. $f(x) = \frac{g(x)}{h(x)}$ $\quad f' = \frac{g'h - gh'}{h^2}$ $f'(2) = \frac{-2(-1) - 3(4)}{(-1)^2}$ $f'(2) = \frac{2-12}{1}$ $f'(2) = -10$	20. $f(x) = g(x)h(x)$ $f'(2) = 14$

TEST PREP

1. D
2. E
3. B
4. D
5. C

FREE RESPONSE **Your score: ____ out of 4**



Graph of g

1. A continuous function g is defined on the closed interval $-8 \le x \le 6$ and is shown above.

(a) Find the approximate value of $g'(4)$. Show the computations that lead to your answer.

$\frac{4-1}{3-5} = \frac{3}{-2}$ or $\frac{f(3) - f(5)}{3-5}$ $\quad g'(4) = -\frac{3}{2}$

1 point for computation $\quad$ 1 point for correct answer

(b) Let h be the function defined by $h(x) = \frac{g(x)}{x^2+1}$. Find $h'(-2)$.

$h'(x) = \frac{g'(x)(x^2+1) - g(x)(2x)}{(x^2+1)^2}$ $\quad h'(-2) = \frac{\left(\frac{4}{3}\right)(5) - (1)(-4)}{25} = \frac{32}{75}$

1 point for derivative set up $\quad$ 1 point for correct answer