

# 3.4 Chain Rule

## CALCULUS

Write your questions here!

Find the derivative.

$$f(x) = (3x + 1)^2$$

### CHAIN RULE

$$\frac{d}{dx} f(g(x)) =$$

Find the derivative of the following.

$$f(x) = (x^2 - 5)^4$$

$$f(x) = \sqrt{4x - 3}$$

$$y = 2(4x - 7)^3$$

$$f(x) = \frac{1}{\sqrt[3]{5x - 3}}$$

Evaluate

$$f(x) = 2x\sqrt{1 - x}$$

Find  $f'(-3)$

Find the derivative.

$$f(x) = \left(\frac{t^2 + 1}{2t - 5}\right)^3$$

Find  $f'(4)$  given the following:

$$g(4) = 4 \text{ and } g'(4) = -2$$

$$h(4) = 9 \text{ and } h'(4) = 5$$

$$f(x) = (g(x))^2$$

$$f(x) = \sqrt{h(x)}$$

$$f(x) = h(g(x))$$

### SUMMARY:

Now,  
summarize  
your notes  
here!



## 3.4 Chain Rule

## PRACTICE

Find the derivative of the following.

1.  $f(x) = (3x^2 - 1)^5$

2.  $f(r) = \sqrt[3]{5r^2 - 2r + 1}$

3.  $y = \frac{1}{(7x^2 - 1)^2}$

4.  $h(x) = 2\sqrt{3x^2 - 5}$

5.  $f(x) = (\pi x - 1)^2 + 7$

6.  $g(x) = 4x - \frac{3}{\sqrt{2x+1}}$

**Find the derivatives of the following.**

7.  $y = x\sqrt{2x - 1}$

8.  $y = (x^3 + e)^{-2}$

9.  $g(x) = 2x(x^3 - 1)^2$

10.  $h(x) = \frac{6x^2 - 5}{\sqrt{2 - 5x}}$

**Evaluate the derivative at a point.**

11.  $f(x) = \sqrt{1 + (x^2 - 1)^3}$   
 $f'(2) =$

12.  $y = \frac{x+1}{\sqrt{2x-1}}$   
 $\frac{dy}{dx}\Big|_{x=1}$

**Write the equation of the tangent line and the normal line at the point given.**

13.  $f(x) = \sqrt{x^2 - 9}$  at  $x = 5$

14.  $f(x) = \frac{1}{(3-2x)^2}$  at  $x = 1$

**Particle Motion**

15. The position of a particle moving along a coordinate line is  $s = \sqrt{1 + 4t}$ , with  $s$  in meters and  $t$  in seconds. Find the particle's velocity at  $t = 6$ .

16. If  $s = \frac{t}{t^2+5}$  is the position function of a moving particle for  $t \geq 0$ , then at what instant of time will the particle start to reverse its direction of motion and where is it at the instant?

**Find  $f'(5)$  given the following.**

17.  $f(x) = g(x) + h(x)$

18.  $f(x) = (h(x))^2$

19.  $f(x) = \sqrt{g(x)}$

20.  $f(x) = 2g(x)h(x)$

**$g(5) = 9$  and  $g'(5) = 6$**

**$h(5) = 5$  and  $h'(5) = -4$**

21.  $f(x) = \frac{1}{h(x)}$

22.  $f(x) = g(h(x))$

## MULTIPLE CHOICE

1. Let  $f(x) = x \cdot g(h(x))$  where  $g(4) = 2$ ,  $g'(4) = 3$ ,  $h(3) = 4$ , and  $h'(3) = -2$ . Find  $f'(3)$ .

- (A)  $-18$
- (B)  $-16$
- (C)  $-7$
- (D)  $7$
- (E)  $11$

2. Let  $m$  and  $b$  be real numbers and let the function  $f$  be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \leq 1 \\ mx + b & \text{for } x > 1 \end{cases}$$

If  $f$  is both continuous and differentiable at  $x = 1$ , then

- (A)  $m = 1, b = 1$
  - (B)  $m = 1, b = -1$
  - (C)  $m = -1, b = 1$
  - (D)  $m = -1, b = -1$
  - (E) none of the above
3. A particle moves on the  $x$ -axis with position defined by:  $x(t) = t^3 - 6t^2 + 2t + 1$  where  $t \geq 0$ . What is the velocity of the particle when its acceleration is zero?
- (A)  $-11$
  - (B)  $-10$
  - (C)  $-1$
  - (D)  $2$
  - (E)  $50$

4. If  $f(x) = \sqrt{1 + \sqrt{x}}$ , find  $f'(x)$ .

- (A)  $\frac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
- (B)  $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$
- (C)  $\frac{1}{4\sqrt{1+\sqrt{x}}}$
- (D)  $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
- (E)  $\frac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$



You are allowed to use a graphing calculator for #5



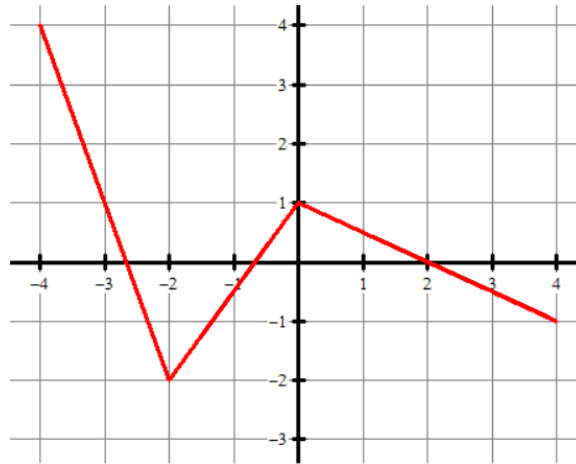
5. If  $(x) = \left(1 + \frac{x}{20}\right)^5$ , find  $f''(40)$ .

- (A) 0.068
- (B) 1.350
- (C) 5.400
- (D) 6.750
- (E) 540.000

**FREE RESPONSE**

Your score: \_\_\_\_ out of 4

1. The graph of the function  $f$ , shown below, consists of three line segments. Suppose  $g(x)$  is a function whose derivative is  $f$ .



Graph of  $f$

(a) Suppose  $y = x + 7$  is the equation for the line tangent to the graph of  $g(x)$  at  $x = -3$ . Let  $h$  be the function defined by  $h(x) = (g(x))^2$ . Find  $h'(-3)$ .

(b) Describe the shape of the graph of  $g(x)$  near  $x = 2$ .

(c) Give a piecewise defined equation for  $g''(x)$ .