6.3 Optimization
Calculus

1. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

2. An open rectangular box is to be made from a 9 × 12 inch piece of tin by cutting squares of side x inches from the corners and folding up the sides. What should x be to maximize the volume of the box?

3. Find the point on the graph of the function \( f(x) = x^2 \) that is closest to the point \((2, \frac{1}{2})\).

4. A rectangle is formed with the base on the x-axis and the top corners on the function \( y = 20 - x^2 \). Find the dimensions of the rectangle with the largest area.

5. What is the smallest product of two numbers given that one number is exactly 8 greater than the other number?
6. A 216 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence right down the middle (parallel to one of the sides).  \( \textbf{(a)} \) What dimensions for the outer rectangle will require the smallest total length of fence?  \( \textbf{(b)} \) How much fence will be needed?

7. Two towers are 45 feet apart. One is 15 feet high and the other is 20 feet high. There is a stake in the ground between the towers. The top of each tower has a wire tied to it that connects to the stake on the ground. Where should the stake be placed to use the least amount of wire?

8. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

---

**Answers to 6.3 CA #1**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24 and 8</td>
<td>2. ( \frac{7-\sqrt{13}}{2} \approx 1.697 ) inches</td>
<td>3. (1, 1)</td>
<td>4. ( 2 \sqrt{\frac{40}{3}} ) meters</td>
</tr>
</tbody>
</table>
| 5. \(-16\) | 6. a) 12 \( \times \) 18 meters  
b) 72 meters | 7. 19.286 feet from the 15-feet tall tower | 8. 300 \( \times \) 600 meters |