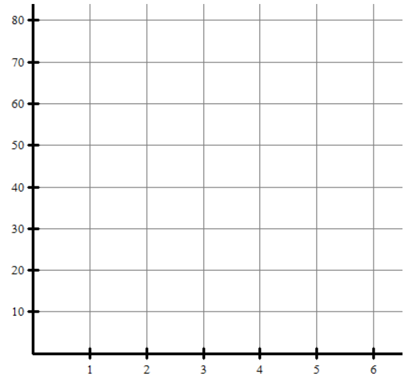
7.1 Rectangular Approximation

NOTES

CALCULUS

Write your questions here!

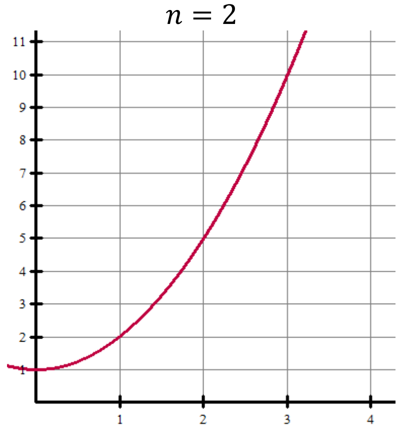
Car travels 60 miles per hour



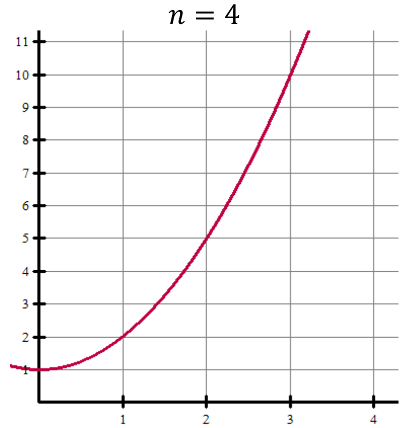
Left Endpoint Rectangle for interval [1,3] with *n* subintervals

$$f(x) = x^2 + 1$$

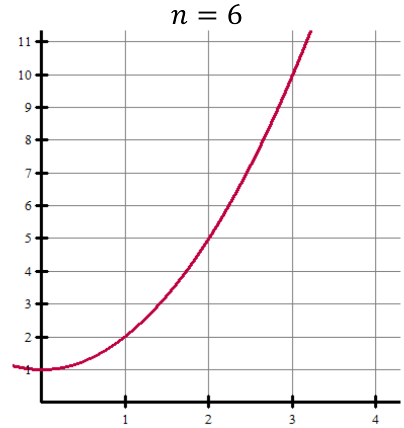
$n = 2$



$n = 4$



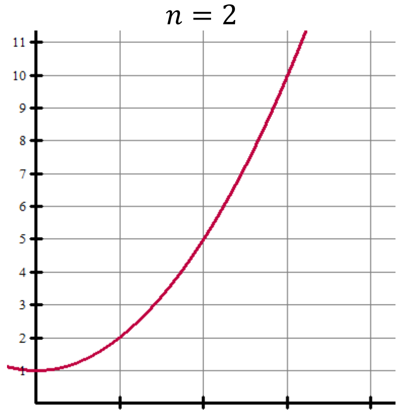
$n = 6$



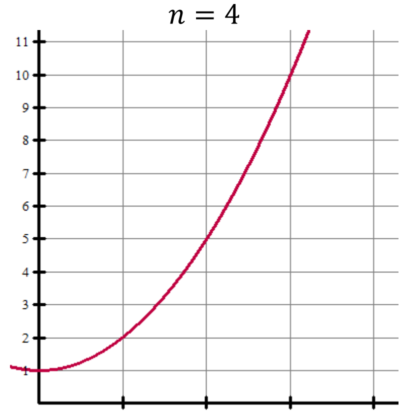
Right Endpoint Rectangle for interval [1,3] with *n* subintervals

$$f(x) = x^2 + 1$$

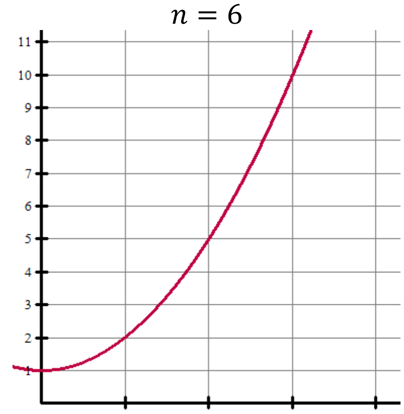
$n = 2$



$n = 4$



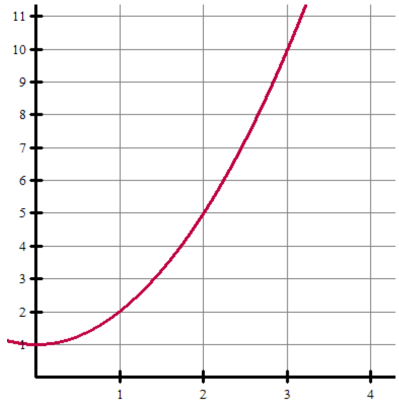
$n = 6$



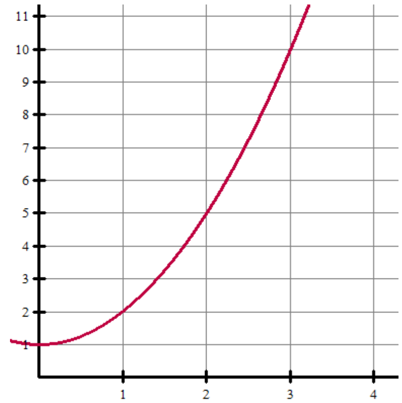
Midpoint Rectangle for interval [1,3] with *n* subintervals

$$f(x) = x^2 + 1$$

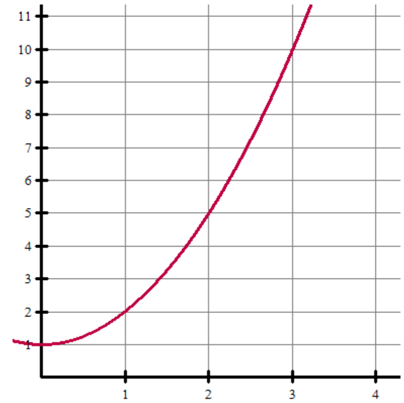
$n = 2$



$n = 4$

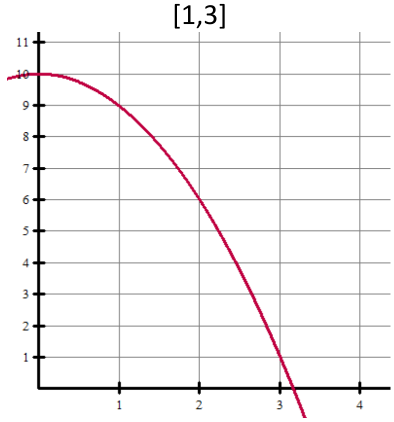


$n = 6$

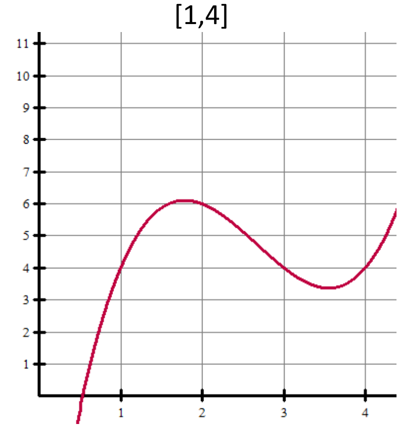


Sketch the following rectangular approximations

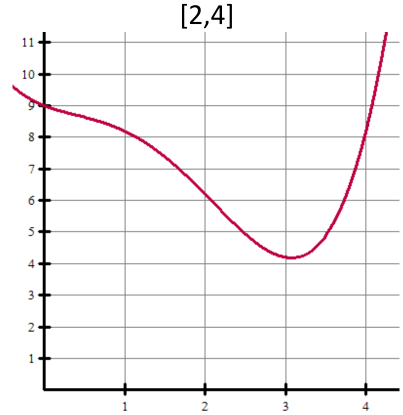
Left Endpoint $n = 5$
[1,3]



Right Endpoint $n = 8$
[1,4]



Midpoint $n = 6$
[2,4]



The rate at which water is being pumped into a tank is given by the continuous and increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 13$ minutes, is given below.

Time (minutes)	0	4	6	10	13
$R(t)$ **(gallons/min)**	7	13	18	23	27

SUMMARY

Now, summarize your notes here!

Use right Riemann Sum with 4 subintervals to approximate the area under the curve.

What does this represent?

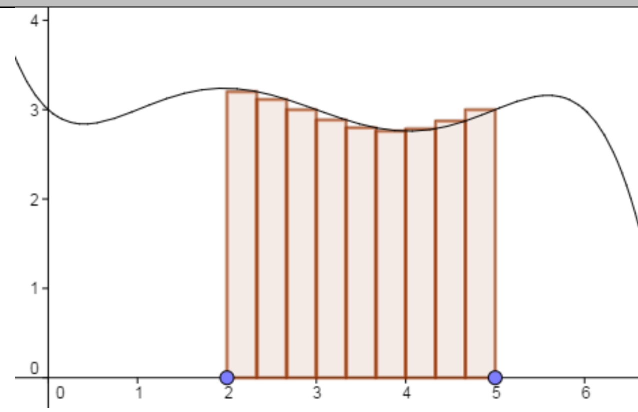
Is the approximation greater or less than the true value?

Use the graph to answer 1-3.

1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

2. Is the approximation less than or greater than the true value?

3. What is the width of each rectangle?



You can use a calculator on 4-13

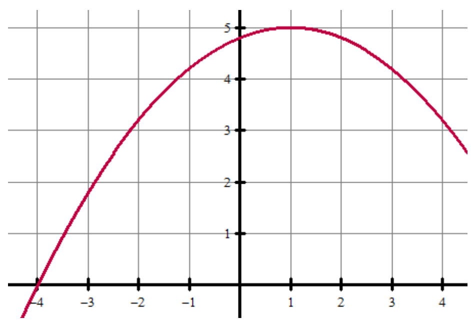
Sketch the following rectangular approximations. Find the width of each subinterval.

4. Midpoint on the interval [1,4] with $n = 6$ subintervals

Width of each subinterval =

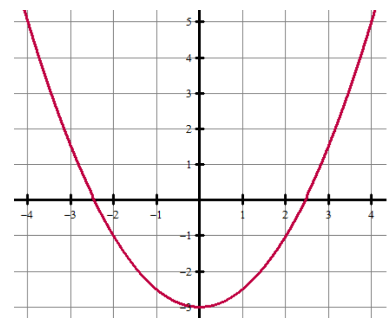
5. Right Endpoint on [-2,2] with $n = 5$ subintervals

Width of each subinterval =



6. Left Endpoint on [-2,4] with $n = 12$ subintervals

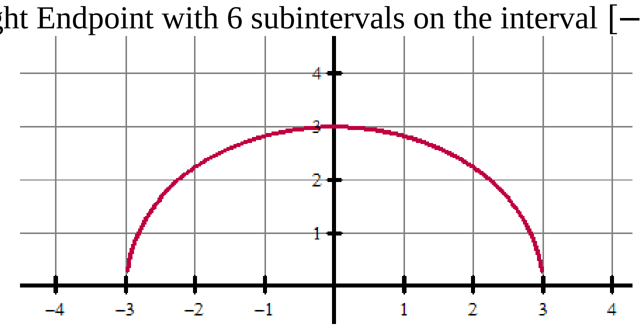
Width of each subinterval =



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

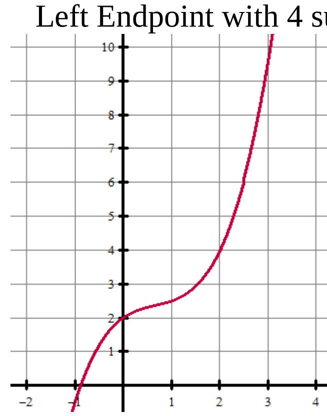
7. $f(x) = \sqrt{9 - x^2}$

Right Endpoint with 6 subintervals on the interval $[-2,1]$



8. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$

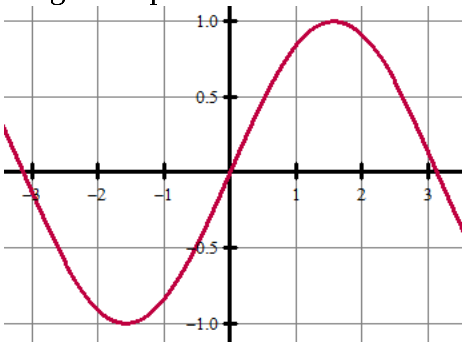
Left Endpoint with 4 subintervals on the interval $[1,3]$



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

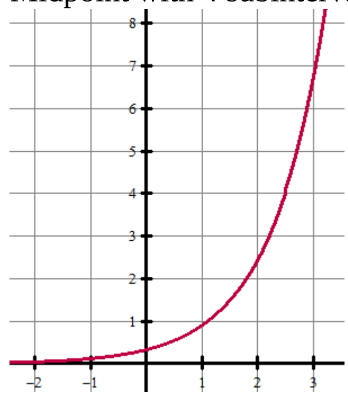
9. $f(x) = \sin x$

Right Endpoint with 3 subintervals on the interval [0,2]



10. $f(x) = \frac{e^x}{3}$

Midpoint with 4 subintervals on the interval [1,3]



Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

b. What does your answer from part (a) represent?

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value?

Use the information provided to answer the following.

12. A rectangular pool gets deeper from one end of the pool to the other. The table shows the depth $h(x)$ of the water at 4 foot intervals from one end of the pool to the other.

position, *x* (feet)	0	4	8	12	16	20	24	28	32
$\boldsymbol{h(x)}$ **(feet)**	6.5	8	9.5	10	11	11.5	12	13	13.5

a. Use the data from the table to find an approximation for $h'(10)$, and explain the meaning of $h'(10)$ in terms of the depth of the pool. Show the computations that lead to your answer.

b. Use a midpoint Riemann Sum with 4 subintervals to approximate the area under the curve.

---

13. Particle *A* moves along a horizontal line with velocity $v(t)$, where $v(t)$ is a positive continuous function of *t*. The time *t* is measured in cm/sec. The velocity of the particle at selected times is given in the table.

t **(sec)**	0	2	5	7	10
$\boldsymbol{v(t)}$ **(cm/sec)**	1.7	6.8	7.4	15.6	24.9

a. Use the data from the table to approximate the distance traveled by a particle *A* over the interval $0 \leq t \leq 10$ seconds by using a left Riemann Sum with four subintervals. Show the computations that lead to your answer.

b. Assuming that $v(t)$ is a continuous increasing function, is the approximation greater or less than the true value?

c. Particle *B* moves along the same horizontal line with position $x(t) = te^{sin3t}$. Which particle is traveling faster at time $t = 5$? Explain your answer.

You can use a calculator on 1-8

MULTIPLE CHOICE

1. A left Riemann Sum with 4 equal subdivisions is used to approximate the area under the sine curve from $x = 0$ to $x = \pi$. What is the approximation?

   (A) $\frac{\pi}{4}\left(0 + \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4}\right)$

   (B) $\frac{\pi}{4}\left(0 + \frac{1}{2} + \frac{\sqrt{3}}{2} + 1\right)$

   (C) $\frac{\pi}{4}\left(0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right)$

   (D) $\frac{\pi}{4}\left(0 + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right)$

   (E) $\frac{\pi}{4}\left(\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 1\right)$

2. A truck moves with positive velocity $v(t)$ from time $t = 3$ to time $t = 15$. The area under the graph of $y = v(t)$ between 3 and 15 gives

   (A) the velocity of the truck at $t = 15$
   (B) the acceleration of the truck at $t = 15$
   (C) the position of the truck at $t = 15$
   (D) the distance traveled by the truck from $t = 3$ to $t = 15$
   (E) the average position of the truck in the interval from $t = 3$ to $t = 15$

3. The first derivative of the function f is given by $f'(x) = \frac{\sin^2 x}{x} - \frac{2}{9}$. How many critical values does f have on the open interval $(0,10)$?

   (A) One
   (B) Two
   (C) Three
   (D) Four
   (E) Six

4. If $y = \sin(x - \sin x)$, what is the smallest positive value of x for which the tangent line is parallel to the x-axis?

   (A) 1.677
   (B) 2.310
   (C) 3.142
   (D) 3.973
   (E) 6.283

5. A particle's height at a time $t \geq 0$ is given by $h(t) = 100t - 16t^2$. What is its maximum height?

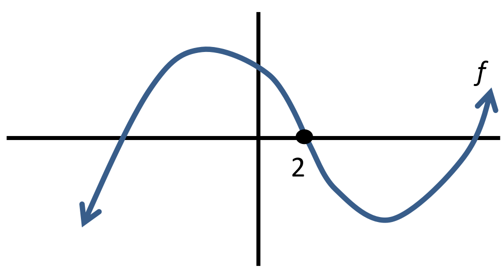
(A) 312.500
(B) 156.250
(C) 78.125
(D) 6.250
(E) 3.125

6. The speeds of a bicyclist at various times *t* are given in the table below. Assume that the bicyclist's acceleration is positive on $(0,3)$ and negative on $(3,6)$. If at $t = 3$ minutes, the bicycle has traveled 1.25 miles, then at $t = 4$ minutes, which of the following could represent the total distance traveled by the bicyclist?

Minutes	0	1	2	3	5	6	7
Miles/hour	0	20	40	45	35	20	5

(A) 1.5 miles
(B) 1.9 miles
(C) 1.25 miles
(D) 2 miles
(E) 1.8 miles

7. The graph of a function, *f*, is shown below. What can be deduced about the function from its graph?



NOTE:
∈ is the "element of" symbol, it is used to show membership in a set of numbers. So, $x \in (-\infty, 2)$ means all values of *x* are in the open interval $(-\infty, 2)$

(A) $f''(x) < 0$ for $x \in (-\infty, 2)$
(B) $f''(2) = 0$
(C) $f''(x) > 0$ for $x \in (2, \infty)$
(D) $x = 2$ is a point of inflection for *f*.
(E) All of the above

8. The table below shows selected values of $f(x)$ and $g(x)$. If $h(x) = g(f(x))$, what is $h'(1)$?

(A) 1
(B) 3
(C) 9
(D) 15
(E) 25

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	1	5
3	5	1	5	3
5	3	5	3	1