UNIT 3 Basic Differentiation

REVIEW

Evaluate the limit.

1. $\lim_{x\to\infty} \frac{\overset{\infty}{x^3} + \cancel{5x^2} - \cancel{x}}{\cancel{1} - e^x} = 0$

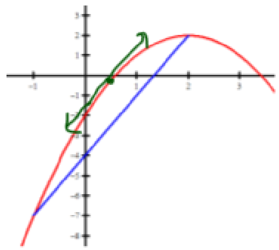
$-e^{\infty}$

big on top / negative REALLY big on bottom

2. $\lim_{x\to 2} \frac{x^2+7x-18}{x^2-2x} = \frac{(x+9)\cancel{(x-2)}}{x\cancel{(x-2)}} = \frac{x+9}{x} = \frac{2+9}{2} = \frac{11}{2}$

Given $f(x)$ on a given interval $[a,b]$, find a value c that satisfies the Mean Value Theorem.

3. $f(x) = -x^2 + 4x - 2;\ [-1, 2]$



$f(-1) = -(-1)^2 + 4(-1) - 2 = -7$

$f(2) = -(2)^2 + 4(2) - 2 = 2$

$(-1,-7)$ and $(2,2)$

$\frac{2-7}{2--1} = \frac{9}{3} = 3$

$f'(x) = -2x + 4$

$3 = -2x + 4$

$-4 \qquad -4$

$-1 = -2x$

$\frac{1}{2} = x$

$c = \frac{1}{2}$

Find b and c so that $f(x)$ is differentiable at $x = 1$.

4. $f(x) = \begin{cases} 3x^2 + 4x, & x \le 1 \\ 2x^3 + bx + c, & x > 1 \end{cases}$

Since it is differentiable it must be continuous!

continuous at x = 1

$3x^2 + 4x = 2x^3 + bx + c$

$3(1)^2 + 4(1) = 2(1)^3 + b(1) + c$

$3 + 4 = 2 + b + c$

$5 - b = c$

differentiable at x = 1

$6x + 4 = 6x^2 + b$

$6(1) + 4 = 6(1)^2 + b$

$10 = 6 + b$

$4 = b$

$5 - 4 = c$

$1 = c$ $\quad$ $4 = b$

Find the derivative of the following.

5. $f(x) = \frac{\sin x}{x^2+1}$ $\quad \begin{matrix} u \\ v \end{matrix}$ $\quad \frac{u'v - uv'}{v^2}$

$u = \sin x \qquad v = x^2 + 1$

$u' = \cos x \qquad v' = 2x$

$$\boxed{f'(x) = \frac{\cos x\,(x^2+1) - \sin x\,(2x)}{(x^2+1)^2}}$$

6. $g(x) = \sqrt{2x^3 - 4x}$

$g(x) = (2x^3 - 4x)^{1/2}$

$g'(x) = \frac{1}{2}(2x^3 - 4x)^{-1/2}(6x^2 - 4)$

$$\boxed{g'(x) = \frac{6x^2-4}{2\sqrt{2x^3-4x}}} \text{ or } \frac{3x^2-2}{\sqrt{2x^3-4x}}$$

7. $y = \frac{x^3+4x-1}{2x}$

$y = \frac{x^3}{2x} + \frac{4x}{2x} - \frac{1}{2x}$

$y = \frac{1}{2}x^2 + 2 - \frac{1}{2}x^{-1}$

$y' = x + \frac{1}{2}x^{-2}$

$$\boxed{y' = x + \frac{1}{2x^2}}$$

8. $h(x) = \cos^2(4x)$

$h(x) = [\cos(4x)]^2$

$h'(x) = 2[\cos(4x)]^1(-\sin(4x))(4)$

$$\boxed{h'(x) = -8\cos(4x)\sin(4x)}$$

Find the following

9. $f(x) = x^2\sin(x)$

$u = x^2$ $\quad v = \sin x$

$f\left(\frac{\pi}{2}\right) =$

$u' v + u v'$ $\quad u' = 2x$ $\quad v' = \cos x$

$f'(x) = 2x\sin x + x^2\cos x$

$f'\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right)^2\cos\left(\frac{\pi}{2}\right)$

$f'\left(\frac{\pi}{2}\right) = \pi \cdot 1 + \left(\frac{\pi}{2}\right)^2 \cdot 0$

$f'\left(\frac{\pi}{2}\right) = \pi$

10. $g(x) = \frac{1}{\sqrt{x}}$

$g''(x) =$

$g(x) = x^{-1/2}$

$g'(x) = -\frac{1}{2}x^{-3/2}$

$g''(x) = \frac{3}{4}x^{-5/2}$

$g''(x) = \frac{3}{4\sqrt{x^5}}$

Write the equation of the tangent line and the normal line at the point given.

11. $f(x) = 3\tan x$ at $x = \pi$

$f'(x) = 3\sec^2 x$

$f'(\pi) = 3\sec^2(\pi)$

$f'(\pi) = 3\left(\frac{1}{-1}\right)^2 = 3$

$f(\pi) = 3\tan(\pi)$

$f(\pi) = 3\left(\frac{0}{1}\right) = 0$

$(\pi, 0)$

tangent line

$y - 0 = 3(x - \pi)$

$y = 3(x - \pi)$

normal line

$y = -\frac{1}{3}(x - \pi)$

Particle Motion

12. The position of a particle moving along a coordinate line is $s(t) = 2t^3 - 6t$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t = 6$.

velocity $= s'(t) = 6t^2 - 6$

$s'(6) = 6(6)^2 - 6 =$ 210 meters per second

acceleration $= s''(t) = 12t$

$s''(6) = 12(6) =$ 72 meters per second^2

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13. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

a) When does the body reverse direction?

when velocity changes signs

$t = 4$ and 8

b) When is the body moving at a constant speed?

from 6-7 seconds

c) What is the body's maximum speed?

Speed = |velocity|

3 meters/sec

d) At what time interval(s) is the body slowing down?

when velocity and acceleration have different signs

velocity is positive (0,4) and (8,9)

velocity is negative (4,8)

acceleration is positive (0,2) and (7,9)

acceleration is negative (2,6)

so, slowing down (2,4) and (7,8)

Use the information to find the following.

14. The table shows the number of stores of a popular US coffee chain from 2000 to 2006. The number of stores recorded is the number at the start of each year, on January 1st.

t (year)	2000	2001	2002	2003	2004	2005	2006
S (stores)	1996	2729	3501	4272	5239	6177	7353

Approximate the instantaneous rate of change in coffee stores per year at the beginning of 2003.

$S'(2003) \approx$ slope of the secant line of two close points 2002 and 2004

$(2002, 3501)$ $(2004, 5239)$ $\qquad \dfrac{5239-3501}{2004-2002} = \dfrac{1738}{2} = \boxed{869 \text{ stores/year}}$

 **You are allowed to use a graphing calculator for #15** 

15. The amount $A(t)$ of pain reliever in milligrams in a patient's system after t minutes is given by $A(t) = 8te^{-t/50}$

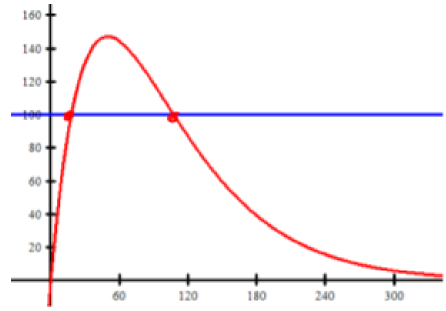
a. Find $A(60)$. Explain what it means in a sentence.

$A(60) = 144.573$ At 60 minutes the patient system has 144.573 mg of pain reliever

b. Find $A'(60)$. Explain what it means in a sentence.

$A'(60) = -0.481$ or -0.482

At 60 minutes the pain reliever in the patient's system is changing -0.481 mg per minute



$t = 17.870$ and 107.6645

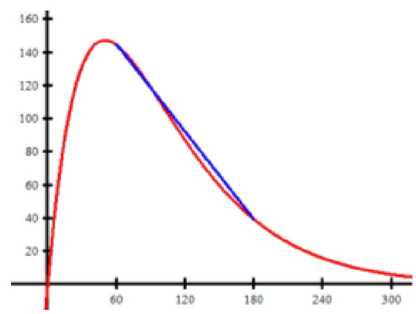
c. Find $A(t) = 100$. Explain what it means in a sentence.

Time that the pain reliever in the patient's system is at 100 mg

d. What is the average rate of change of students from 60 minutes to 180 minutes?

$(60, 144.573)$ $(180, 39.346)$

$\dfrac{39.346-144.573}{180-60} = \dfrac{-105.227}{120} = \boxed{-0.876 \text{ mg/min}}$



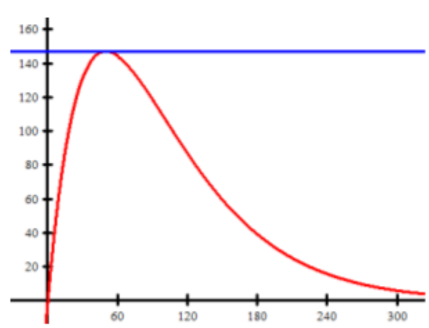
e. What is the instantaneous rate of change at 180 minutes?

$f'(180) = -0.568$ mg/min

$t = 50$ minutes

f. When does $A'(t) = 0$? What is happening at this point?

Slope of the tangent line is zero. This is a max or min point.
In this case, it is the maximum pain reliever in the patient's system.



g. Find $\lim_{t\to\infty} A(t)$. Explain what it means in a sentence.

$\lim_{t\to\infty} A(t) \approx 0$

As time approaches infinity, the pain reliever in the patient's system approaches zero.

TEST PREP

1. A particle is traveling along the x-axis. Its position is given by $x(t) = \frac{1-t^2}{t+3}$ at time $t \geq 0$. Find the instantaneous rate of change of x with respect to t when $t = 1$.

u v

$\frac{u'v-uv'}{v^2}$ $u = 1-t^2$ $v = t+3$

$u' = -2t$ $v' = 1$

B

(A) -2
(B) $-\frac{1}{2}$
(C) 0
(D) $\frac{1}{2}$
(E) 2

$x'(t) = \frac{-2t(t+3)-(1-t^2)(1)}{(t+3)^2}$

$x'(1) = \frac{-2(1)(1+3)-(1-1^2)}{(1+3)^2}$

$x'(1) = \frac{-8}{16} = -\frac{1}{2}$

2. The line $2x - y = 9$ is tangent to the curve $f(x)$ at the point $(4, -1)$. What is the value of $f'(4)$?

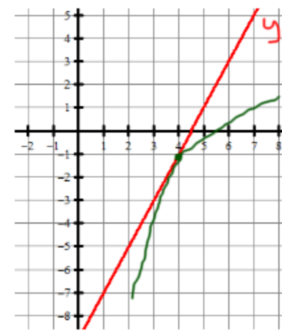
Since 2x - y = 9 is tangent to the f(x) at x = 4,
then the slope of the line 2x - y = 9 is the derivative of f(x) at x = 4

C

(A) -2
(B) $\frac{1}{2}$
(C) 2
(D) 4
(E) 9

$2x-y=9$
$-2x$ $-2x$
$-y = 9-2x$
$y = 2x-9$

$f'(4) = 2$



3. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

E

(A) $\lim\limits_{h\to 0} \frac{e^{x+h}}{h}$
(B) $\lim\limits_{h\to 0} \frac{e^{x+h} - e^e}{h}$
(C) $\lim\limits_{h\to 0} \frac{e^{e+h} - e}{h}$
(D) $\lim\limits_{h\to 0} \frac{e^{e+h} - 1}{h}$
(E) $\lim\limits_{h\to 0} \frac{e^{e+h} - e^e}{h}$

definition of derivative is $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

so, the $f(x) = e^x$, plug that bad boy into the definition of derivative

$\lim\limits_{h\to 0} \frac{e^{x+h}-e^x}{h}$

now, you want to find f'(e), so plug e in for x

$\lim\limits_{h\to 0} \frac{e^{e+h}-e^e}{h}$

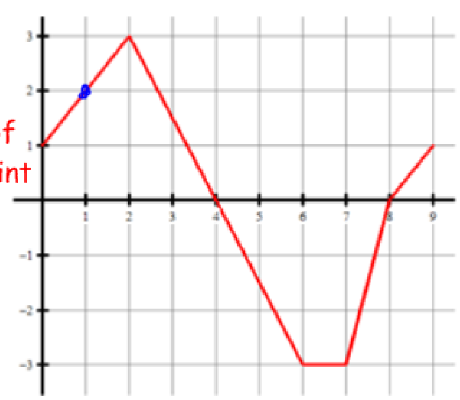
4. The graph of $f(x)$ is shown below. What is the value of $f(1) + f'(1) + 2f'(4)$?

A

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$f(1) = 2$
$f'(1) = 1$
$f'(4) = -\frac{3}{2}$

derivative is the slope of the tangent line at the point



$f(1) + f'(1) + 2f'(4)$

$2 + 1 + 2\left(-\frac{3}{2}\right)$

$3 - 3 = 0$