TEST PREP

1. What are the $x$-coordinate(s) of the points of inflection for the graph of $f(x) = \sin^2 x$ on the closed interval $[0, \pi]$?

(A) $x = \frac{3\pi}{4}$ only
(B) $x = \frac{\pi}{4}, x = \frac{\pi}{2}$, and $x = \frac{3\pi}{4}$
(C) $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$
(D) $x = \frac{\pi}{2}$ only
(E) $x = \frac{\pi}{4}$ only

2. The function defined by $g(x) = 4x^3 - 3x^2$ for all values of $x$ has a relative maximum at $x =$

(A) $-\frac{1}{2}$
(B) $0$
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$
(E) $1$

3. The graph of the derivative of function $f$ is shown below. At what value of $x$ does function $f$ have a relative maximum?

(A) 1
(B) $-1$
(C) 0
(D) $-2$
(E) 3

4. The function $g$ is defined by the equation $g(x) = 6x^5 - 10x^3$. Determine the values of $x$ for which the graph of function $g$ is concave upwards.

(A) $x > \frac{1}{2}$
(B) $-\frac{\sqrt{2}}{2} < x < 0$ or $x > \frac{\sqrt{2}}{2}$
(C) $-\frac{1}{2} < x < 0$ or $x > \frac{1}{2}$
(D) $-\frac{1}{2} < x < \frac{1}{2}$
(E) $-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$
5. For what values of \( k \) will \( f(x) = x^2 + \frac{k}{x} \) have a relative minimum at \( x = 2 \)?

(A) \(-2\)
(B) \(2\)
(C) \(8\)
(D) \(-16\)
(E) \(16\)

\[
f(x) = x^2 + \frac{k}{x} \quad f'(x) = 2x - \frac{k}{x^2} \quad f''(x) = 2 + \frac{2k}{x^3}
\]

\[
2x - \frac{k}{x^2} = 0 \quad \Rightarrow \quad x\left(2 - \frac{k}{x^3}\right) = 0
\]

\[
x = \frac{k}{2}
\]

\[
y = \left(\frac{k}{2}\right)^2 = 16 = k
\]

6. The graph shown below shows the derivative \( f'(x) \) of the function \( f(x) \). At what value(s) of \( x \) does function \( f \) have a point of inflection?

(A) \(c\) and \(e\) only
(B) \(a, b, c,\) and \(d\) only
(C) \(a\) and \(c\) only
(D) \(b\) and \(d\) only
(E) \(a\) only

7. An equation of the line tangent to the graph of \( f(x) = 2x^3 - 3x^2 \) at its point of inflection is

(A) \(3x + 2y = 5\)
(B) \(6x + 4y = 1\)
(C) \(6x + 4y = 5\)
(D) \(3x + 2y = 1\)
(E) \(6x - 4y = 1\)

\[
f(x) = 2x^3 - 3x^2 \quad f'(x) = 6x^2 - 6x \quad f''(x) = 12x - 6
\]

\[
12x - 6 = 0 \quad \Rightarrow \quad x = \frac{1}{2}
\]

\[
f'\left(\frac{1}{2}\right) = 6 \left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) = -\frac{3}{4}
\]

\[
f''\left(\frac{1}{2}\right) = 12 \cdot \frac{1}{2} - 6 = 0
\]

\[
y = f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 = \frac{1}{2}
\]

8. The graph of the function \( y = f(x) \) is shown below. On which of the following intervals is \( f'(x) > 0 \) and \( f''(x) > 0 \)?

(A) I, II, and III
(B) II and III only
(C) II only
(D) I only
(E) III only

I. \(c < x < d\)
II. \(a < x < b\)
III. \(b < x < c\)
9. The graph of the derivative of function $f$ is shown below. Where on the interval $[-2, 3]$ is function $f$ decreasing?
   (A) $[-2, 3]$
   (B) $[-1, 1]$
   (C) $[1, 3]$
   (D) $[-2, 0]$
   (E) $[0, 3]$

10. For what interval is $f(x) = \frac{1}{1-x^2}$ increasing?
   (A) Function $f$ increases for all real values of $x$
   (B) $(-\infty, -1) \cup (-1, 0]$
   (C) $[0, 1) \cup (1, \infty)$
   (D) $(-1, 1)$
   (E) $(-\infty, -1) \cup (1, \infty)$

11. The table below shows various values for the derivatives of differentiable functions $f, g,$ and $h$. Which of these functions must have a relative maximum on the open interval $(-3, 3)$?
   (A) $g$ only
   (B) $f, g,$ and $h$
   (C) $g$ and $h$ only
   (D) $h$ only
   (E) $f$ only

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>$g'(x)$</td>
<td>-1.5</td>
<td>-1</td>
<td>-0.5</td>
<td>0</td>
<td>0.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>$h'(x)$</td>
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<td>0</td>
<td>-0.5</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
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</tr>
</tbody>
</table>

12. If \[ \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = 2.637 \] then the graph of function $f$ at $x = -2$ is
   (A) Decreasing
   (B) Concave downwards
   (C) Increasing
   (D) Concave upwards
   (E) Stationary

13. The graph of $y = f(x)$ is shown below. If $f$ is twice-differentiable, which of the following is true?
   (A) $f(x) < 0$, $f'(x) < 0$, $f''(x) < 0$
   (B) $f(x) > 0$, $f'(x) < 0$, $f''(x) > 0$
   (C) $f(x) > 0$, $f'(x) > 0$, $f''(x) > 0$
   (D) $f(x) > 0$, $f'(x) < 0$, $f''(x) < 0$
   (E) $f(x) > 0$, $f'(x) > 0$, $f''(x) < 0$