Recall: What is a removable discontinuity?

\[ \lim_{x \to c} f(x) \text{ exists, but } \lim_{x \to c} f(x) \neq f(c) \]

1. \[ f(x) = \frac{x^2-1}{x-1} \]

Find the \( x \)-value of the hole. How do we find the \( y \)-value?

2. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2+6x+8}{x+4} \) when \( x \neq -4 \), then \( f(-4) = \)

3. Let \( f \) be the function defined by \[ f(x) = \begin{cases} \frac{x^2-3x-18}{x-6}, & x \neq 6 \\ a, & x = 6 \end{cases} \]. For what value of \( a \) is \( f \) continuous at \( x = 6 \)?

1.13 Removing Discontinuities

1. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2-9}{x-3} \) when \( x \neq 3 \), then \( f(3) = \)

2. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2+8x-20}{x+10} \) when \( x \neq -10 \), then \( f(-10) = \)
3. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2 - 5x + 4}{x - 1} \) when \( x \neq 1 \), then \( f(1) = \)

4. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2 + 14x + 48}{x + 8} \) when \( x \neq -8 \), then \( f(-8) = \)

5. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 - 2x - 15}{x - 5}, & x \neq 5 \\
\frac{a}{x}, & x = 5 
\end{cases}
\]
For what value of \( a \) is \( f \) continuous at \( x = 5 \)?

6. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 - 16x + 63}{x - 7}, & x \neq 7 \\
\frac{b}{x}, & x = 7 
\end{cases}
\]
For what value of \( b \) is \( f \) continuous at \( x = 7 \)?

7. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 - 8x}{x}, & x \neq 0 \\
\frac{c}{x}, & x = 0 
\end{cases}
\]
For what value of \( c \) is \( f \) continuous at \( x = 0 \)?

8. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 - 8x + 15}{x - 3}, & x \neq 3 \\
\frac{a}{x}, & x = 3 
\end{cases}
\]
For what value of \( a \) is \( f \) continuous at \( x = 3 \)?

9. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 + 5x + 4}{b(x + 1)}, & x \neq -1 \\
\frac{b}{x}, & x = -1 
\end{cases}
\]
For what value of \( b \) is \( f \) continuous at \( x = 1 \)?

10. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{x^2 - 49}{c(x + 7)}, & x \neq -7 \\
\frac{c}{x}, & x = -7 
\end{cases}
\]
For what value of \( c \) is \( f \) continuous at \( x = -7 \)?
11. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{\sin(6x)}{5x}, & x \neq 0 \\
a, & x = 0 
\end{cases}
\]
For what value of \( a \) is \( f \) continuous at \( x = 0 \)?

12. Let \( f \) be the function defined by
\[
f(x) = \begin{cases} 
\frac{5\sin(3x)}{4x}, & x \neq 0 \\
b, & x = 0 
\end{cases}
\]
For what value of \( b \) is \( f \) continuous at \( x = 0 \)?

1.13 Removing Discontinuities

13. Let \( y = \frac{x^2+4x-21}{x^2-9} \). This function has a hole. What is the \( y \)-value of the hole?

(A) \( \frac{5}{3} \)  \hspace{1cm} (B) 3  \hspace{1cm} (C) \( -\frac{10}{3} \)  \hspace{1cm} (D) 0  \hspace{1cm} (E) -3

14. For what value of \( k \) will the function \( f(x) = \frac{x^2-(k+2)x+6}{x-k} \) have a point discontinuity at \( x = k \)?

(A) \( k = -1 \)  \hspace{1cm} (B) \( k = 0 \)  \hspace{1cm} (C) \( k = 1 \)  \hspace{1cm} (D) \( k = 2 \)  \hspace{1cm} (E) \( k = 3 \)