Area and Volume
ANSWERS
AP Calculus Free Response Problems
2002 – current year

2002 Form A #1 [calculator allowed]

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

$2 \begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2\right) dx$

$= 7.515\pi$ or 23.609

$4 \begin{cases} 1: \text{ limits and constant} \\ 2: \text{ integrand} \\ \quad < -1 > \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \quad k\int_a^b \left(R(x)^2 - r(x)^2\right) dx \\ 1: \text{ answer} \end{cases}$

(c) $h'(x) = f'(x) - g'(x) = e^x - \dfrac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$

$h(0.5) = 2.3418$

$h(1) = 2.718$

The absolute minimum is 2.330.

The absolute maximum is 2.718.

$3 \begin{cases} 1: \text{ considers } h'(x) = 0 \\ 1: \text{ identifies critical point} \\ \quad \text{and endpoints as candidates} \\ 1: \text{ answers} \end{cases}$

Note: Errors in computation come off the third point.

2002 Form B #1 [calculator allowed]

Region R

$$\frac{x^3}{1+x^2} = 4 - 2x \text{ at } x = 1.487664 = A$$

1 : Correct limits in an integral in (a), (b), or (c).

(a) $$\text{Area} = \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$$

$$= 3.214 \text{ or } 3.215$$

2 { 1 : integrand; 1 : answer }

(b) Volume

$$= \pi \int_0^A \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2 \right) dx$$

$$= 31.884 \text{ or } 31.885 \text{ or } 10.149\pi$$

3 { 2 : integrand and constant; < −1 > each error; 1 : answer }

(c) $$\text{Volume} = \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$$

$$= 8.997$$

3 { 2 : integrand; < −1 > each error; note: 0/2 if not of the form $k\int_c^d (f(x) - g(x))^2\, dx$; 1 : answer }

2003 Form A #1 [calculator allowed]

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

1: Correct limits in an integral in (a), (b), or (c)

(a) $$\text{Area} = \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx = 0.442 \text{ or } 0.443$$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $$\text{Volume} = \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2 \right) dx = 0.453\pi \text{ or } 1.423 \text{ or } 1.424$$

$$3 : \begin{cases} 2 : \text{integrand} \\ \quad < -1 > \text{ reversal} \\ \quad < -1 > \text{ error with constant} \\ \quad < -1 > \text{ omits 1 in one radius} \\ \quad < -2 > \text{ other errors} \\ 1 : \text{answer} \end{cases}$$

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

$$\text{Volume} = \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$$

$$3 : \begin{cases} 2 : \text{integrand} \\ \quad < -1 > \text{ incorrect but has } \sqrt{x} - e^{-3x} \text{ as a factor} \\ 1 : \text{answer} \end{cases}$$

2003 Form B #1 [calculator allowed]

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$

$f(3) = 36 - 27 = 9$

Tangent line at $x = 3$ is

$y = -3(x - 3) + 9 = -3x + 18$,

which is the equation of line ℓ.

2 :
- 1 : finds $f'(3)$ and $f(3)$
- 1 : finds equation of tangent line or shows (3,9) is on both the graph of f and line ℓ

(b) $f(x) = 0$ at $x = 4$

The line intersects the x-axis at $x = 6$.

$$\text{Area} = \frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right) dx = 7.916 \text{ or } 7.917$$

OR

$$\text{Area} = \int_3^4 \left((18 - 3x) - \left(4x^2 - x^3\right)\right) dx + \frac{1}{2}(2)(18 - 12) = 7.916 \text{ or } 7.917$$

4 :
- 2 : integral for non-triangular region
  - 1 : limits
  - 1 : integrand
- 1 : area of triangular region
- 1 : answer

(c) $$\text{Volume} = \pi \int_0^4 \left(4x^2 - x^3\right)^2 dx = 156.038\pi \text{ or } 490.208$$

3 :
- 1 : limits and constant
- 1 : integrand
- 1 : answer

2004 Form A #2 [calculator allowed]

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$ $= \int_0^1 \left(2x(1-x) - 3(x-1)\sqrt{x}\right) dx = 1.133$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi\int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2\right) dx$ $= \pi\int_0^1 \left(\left(2 - 3(x-1)\sqrt{x}\right)^2 - (2 - 2x(1-x))^2\right) dx$ $= 16.179$	$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$ $\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2004 Form B #1 [calculator allowed]

(a) Area $= \int_1^{10} \sqrt{x-1}\, dx = 18$	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right) dx$ $= 212.057$ or 212.058	$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(c) Volume $= \pi \int_0^{3} \left(10 - \left(y^2 + 1\right)\right)^2 dy$ $= 407.150$	$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

2005 Form A #1 [calculator allowed]

$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$. f and g intersect when $x = 0.178218$ and when $x = 1$. Let $a = 0.178218$.	
(a) $\int_0^a (g(x) - f(x))\,dx = 0.064$ or 0.065	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $\int_a^1 (f(x) - g(x))\,dx = 0.410$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\pi \int_a^1 \left((f(x)+1)^2 - (g(x)+1)^2 \right) dx = 4.558$ or 4.559	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

2005 Form B #1 [calculator allowed]

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_0^S (f(x) - g(x))\, dx$ $= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$ $= 0.429$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx$ $= \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$ $= 4.266$ or 4.267	$3 : \begin{cases} 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \qquad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx$ $= \int_0^S \frac{\pi}{2}\left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx$ $= 0.077$ or 0.078	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2006 Form A #1 [calculator allowed]

$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

2006 Form B #1 [calculator allowed]

For $x < 0$, $f(x) = 0$ when $x = -1.37312$.
Let $P = -1.37312$.

(a) Area of $R = \int_P^0 f(x)\,dx = 2.903$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_P^0 \left((f(x) + 2)^2 - 4 \right) dx = 59.361$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$.

The graph of f and line ℓ intersect at $A = 3.38987$.

Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x\right) - f(x) \right) dx$

$3 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

2007 Form A #1 [calculator allowed]

$\frac{20}{1+x^2} = 2$ when $x = \pm 3$	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961$ or 37.962	2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_{-3}^{3} \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$	3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \frac{\pi}{2} \int_{-3}^{3} \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$ $= \frac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$	3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2007 Form B #1 [calculator allowed]

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R

$= 2.06016 -$ Area of $R = 1.546$

OR

$$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P) \cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2 \right) dx$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{constant and limits} \end{cases}$

2008 Form A #1 [calculator allowed]

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2008 Form B #1 [calculator allowed]

The graphs of $y = \sqrt{x}$ and $y = \dfrac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\displaystyle\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\displaystyle\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\displaystyle\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$

$\displaystyle= \frac{207\pi}{5} = 130.061$ or 130.062

$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\displaystyle\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

2009 Form A #4 [no calculator!]

(a) Area $= \int_0^2 \left(2x - x^2\right) dx$

$$= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2}$$

$$= \frac{4}{3}$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx$

$$= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_{x=0}^{x=2}$$

$$= \frac{4}{\pi}$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

2009 Form B #4 [no calculator!]

(a) Area $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_{x=0}^{x=4} = \frac{4}{3}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4}\right) dx$

$= \frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12}\Big|_{x=0}^{x=4} = \frac{8}{15}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right) dx$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

2010 Form A #4 [no calculator]

(a) Area $= \int_0^9 (6 - 2\sqrt{x})\, dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big\vert_{x=0}^{x=9} = 18$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi\int_0^9 \left((7 - 2\sqrt{x})^2 - (7-6)^2\right) dx$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$
(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$. Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$. Volume $= \int_0^6 \frac{3}{16}y^4\, dy$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2010 Form B #1 [no calculator]

	1 : Correct limits in an integral in (a), (b), or (c)
(a) $\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or 6.817	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right)dx$ $= 168.179$ or 168.180	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^2 (6 - 4\ln(3 - x))^2\,dx = 26.266$ or 26.267	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2011 Form A #3 [calculator allowed]

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b) Area $= \int_0^{1/2} (g(x) - f(x))\, dx$

$= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$

$= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$

$= -\frac{1}{8} + \frac{1}{\pi}$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx$

$= \pi\int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

2011 Form B #3 [calculator allowed]

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2}\cdot 2\cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

2012 #2 [calculator allowed]

$\ln x = 5 - x \quad \Rightarrow \quad x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $$\text{Area} = \int_0^B \left(5 - y - e^y\right) dy = 2.986 \ (\text{or } 2.985)$$

OR

$$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx = 2.986 \ (\text{or } 2.985)$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$$

$3 : \begin{cases} 2 : \text{integrands} \\ 1 : \text{expression for total volume} \end{cases}$

(c) $$\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

2013 #5 [no calculator]

(a) $$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) $$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

2014 #2 [calculator allowed]

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$ Volume $= \pi\int_0^{2.3}\left[(4+2)^2 - (f(x)+2)^2\right]dx$ $= 98.868$ (or 98.867)	$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \int_0^{2.3}\frac{1}{2}(4 - f(x))^2\,dx$ $= 3.574$ (or 3.573)	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^k (4 - f(x))\,dx = \int_k^{2.3}(4 - f(x))\,dx$	$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$

2015 #2 [calculator allowed]

(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$. $\text{Area} = \int_0^A [g(x) - f(x)]\,dx + \int_A^2 [f(x) - g(x)]\,dx$ $= 0.997427 + 1.006919 = 2.004$	$4 : \begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$
(b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2\,dx = 1.283$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $h(x) = f(x) - g(x)$ $h'(x) = f'(x) - g'(x)$ $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)	$2 : \begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2016 #5 No calculator

(a) $\text{Average radius} = \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right) dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$ $= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(b) $\text{Volume} = \pi\int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2 dh = \frac{\pi}{400}\int_0^{10}\left(9 + 6h^2 + h^4\right) dh$ $= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$ $= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}\ \text{in}^3$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$ $-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec	$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2017 – NONE WERE ON THE EXAM THIS YEAR!!