1^{st} Semester
AP Calculus AB Final Exam Topics

<u>45 Multiple Choice Questions total:</u>
28 Non-Calculator
17 Calculator

<u>Limits- 2 Questions</u>

- Limits of Piecewise functions at the changing point
- Strategies for finding limits:
  - BOBOBOTN EATS DC (rational functions)
  - Try to factor, cancel, and then substitute

<u>Continuity/Differentiability- 5 Questions</u>

- Rules for differentiability
  - Right-hand and Left-hand derivatives (slopes) must be the same
  - NO: cusps, vertical tangents, discontinuities
- Rules for Continuity
  - Graph can be drawn without lifting pencil
    - NO: holes or asymptotes
- Know how to evaluate continuity/differentiability of piecewise functions
- Know how to interpret limit notations when dealing with continuity and differentiability

<u>Tangent lines/slopes- 6 Questions</u>

- Write an equation for a tangent line given:
  - f(x) and a point or x-value
  - graph of f'(x) and a point
    - f'(a) = slope (look at the y value on the graph!)
- Find a tangent line parallel to another line
- Find the point where two functions have parallel tangents (set derivatives equal)
- Find the point where the slope of f(x) = a specific value.

<u>Average Rate of Change on an interval (1 Question)</u>

- Do not use the derivative. Use the formula:
  - $\frac{f(b)-f(a)}{b-a}$

<u>Velocity/Acceleration- 2 Questions</u>

- Know how to find v(t) and a(t) given s(t). Also, be able to find where the velocity or acceleration are equal to zero.
- Know how to find the maximum velocity or acceleration

<u>Derivatives/Rules for Derivatives- 10 Questions</u>

- Know all rules for differentiation (formulas AND basics, i.e. constant multiple rule)
  - Emphasis on trig functions, exponential and logarithmic functions
- DON'T FORGET:
  - Chain Rule
  - Product Rule
  - Quotient Rule
- Know how to evaluate a derivative at a point
- "Instantaneous Rate of Change" = slope = derivative

Implicit Differentiation- 2 Questions

- Use when you cannot solve for y.
- Differentiate with respect to x
    - *Always* write $\frac{dy}{dx}$ after you differentiate any term with a "y"

Comparing f, f', and f'' (including finding max/min/inc/dec/concavity/POI)- 13 Questions

- Find maximums, minimums, and critical points given a graph of f'
- Find inflection points given f''(x) factored
- CIPPMXMXIP
    - First derivative tells you: increasing and decreasing intervals, Max/Mins
    - Second derivative tells you: concave up and down intervals, Points of Inflection
        - Find critical points (where f'/f'' = 0 or undefined), make a sign chart.

Related Rates- 3 Questions

- Differentiate all variables (rate you know and want to know) with respect to *t*.
- Know Circumference/Area of a circle
- Know Area of a triangle, Pythagorean Thm, etc.

Optimization- 1 Question

- Finding the max/min given some conditions. Make sure you only differentiate one variable
- Know how to maximize a product of two numbers

Tips for the Calculator Test (17 Questions):

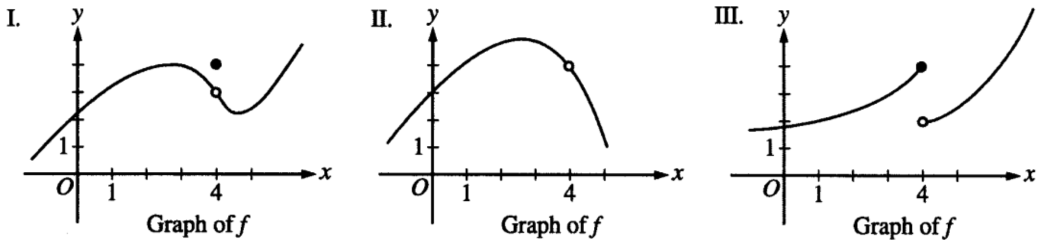
- Use nderiv(function, x, value) to find the derivative of any function at a point
- Graph the derivative of a function using y = nderiv(function, x, x)
- When in doubt, look at a graph
- Instead of trying to solve a difficult equation, to find where a function (or derivative) equals a certain value, calculate the intersection of:
  Y1 = function
  Y2 = value you want function to be equal to
- Know how to calculate Zeros, Maximums, Minimums, and Intersections on the calculator
- Remember to adjust your window and table to fit what you are looking for

AP Calculus AB Name________________________________

Fall Final Review Date________________ Hour____________

CALCULATOR REVIEW

1. For which of the following does $\lim_{x \to 4} f(x)$ exist?



2. What is the average rate of change of y = $\frac{\cos x}{x^2+x+2}$ on the closed interval [-2, 2]?

3. $\lim_{x \to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$

4. If $\lim_{x \to c} f(x) = -\frac{1}{2}$ and $\lim_{x \to c} g(x) = \frac{2}{3}$, find $\lim_{x \to c} \frac{f(x)}{g(x)}$.

5. An object is dropped from the top of a tower. Its height, in meters, above the ground after *t* seconds is given by the equation $y = 300 - 4.9t^2$. Give answers with correct units.

(a) What is the height of the object after 3 seconds?

(b) What is the average speed of the object over the first 3 seconds?

(c) What is the instantaneous speed of the object at 3 seconds?

(d) Write the equation of the tangent line to the graph of *y* when *t* = 3.

6. A particle moves along the *x*-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 2 + 4.1\cos(0.8t)$. What is the acceleration of the particle at time *t* = 3?

7. If $f(x) = \ln(x + 4 + e^x)$, then $f'(0)$ is?

8. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$

9. The function f is continuous on [-2, 2] and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

10. A rock is thrown straight into the air. Its height, in meters, above the ground after t seconds is given by the equation $s(t) = 32t - 4.9t^2$. Show your work and give answers with correct units.

(a) What is the height of the rock after 3 seconds?

(b) What is the average velocity of the rock over the first 3 seconds?

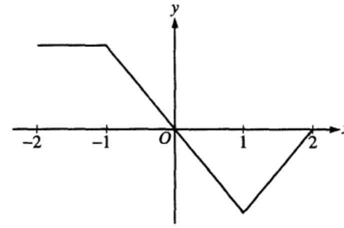
(c) What is the instantaneous velocity of the rock at 3 seconds?

(d) What is the maximum height of the object and how long does it take to fall back to the ground?

11. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at (x, f(x)) equal to 4?

12. The graph of f', the derivative of the function f, is shown to the right. Which of the following statements is true?

(A) f is decreasing for $-1 \le x \le 1$.
(B) f is increasing for $1 \le x \le 2$.
(C) f is not differentiable at $x = -1$ and $x = 1$.
(D) f is increasing for $-2 \le x \le 0$.
(E) f has a local minimum at $x = 0$.



Graph of f'

13. Let f be a differentiable function such that f(3) = 2 and f'(3) = 5. If the tangent line to the graph of f at x = 3 is used to find an approximation to a zero of f, find that approximation.

14. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

(a) On what intervals is f(x) increasing?

(b) On what intervals is the graph of f(x) concave downward?

(c) Find the value of k for which f(x) has 11 as its relative minimum.

15. The radius of a circle is increasing at a constant rate of 0.4 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 25π meters?

16. In the right triangle with a hypotenuse of 13, if θ increases at a constant rate of 2 radians per minute, at what rate is x (the side opposite θ) increasing in units per minute when x equals 5 units?

17. Let f be the function with derivative given by $f'(x) = \cos(x^2 + 1)$. How many relative extrema does f have on the interval $1 < x < 5$?

18. The position of a particle moving along the x-axis is given by the function $x(t) = e^t + t\,e^t$. What is the average velocity of the particle from time t = 0 to time t = 5?

19. Consider the curve defined by $-8x^2 + 5xy + y^3 = -125$

(a) Find dy/dx.

(b) Write an equation for the line tangent to the curve at (3, -1).

(c) There is a number k such that (3.2, k) is on the curve. Using the tangent line in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k such that (3.2, k) is on the curve.

(e) Solve the equation in part (d) for the value of k.

20. If $y = 5^x + 4x - 2$, Find dy/dx.

NON-CALCULATOR REVIEW

1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$

2. What are all the horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the *xy*-plane?

3. $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}$

4. $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$

5. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$

6. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then f(-2) =

7. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4 \end{cases}$ then $\lim_{x\to 2} f(x)$ is

8. Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5 \end{cases}$

   (a) Is f continuous at x = 3? Explain why or why not.

   (b) Find the average rate of change of *f*(*x*) on the closed interval [0, 3].

   (c) Suppose the function g is defined by $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5 \end{cases}$ where *k* and *m* are constants. If g is continuous at *x* = 3, what is the value of *k* when *m* = 2?

9. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

10. $\lim_{x\to\infty} \frac{4(x^3-5x^2+x-4)}{4x^3-3x^2+5x-3}$

11. If the line tangent to the graph of the function f at the point (1, 7) passes through the point (-2, -2), then $f'(1)$ is

12. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at x = -1 is

13. If $(x) = \sqrt{2x}$, then $f'(2) =$

14. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

15. If $= \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

16. A particle moves along a line so that its position, in meters, at any time $t \geq 0$, in seconds, is given by $s(t) = 2t^3 - 11t^2 + 12t - 13$. Show your work and give answers with correct units.

   (a) Write the velocity of the particle as a function of time, t.

   (b) Write the acceleration of the particle as a function of time, t.

   (c) When is the particle at rest? What is its acceleration at these times?

   (d) When does the particle change direction? Justify your answer.

17. The graph of y = -5/(x-2) is concave downward for which values of x?

18. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at x = ?

19. If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of f has inflection points when x = ?

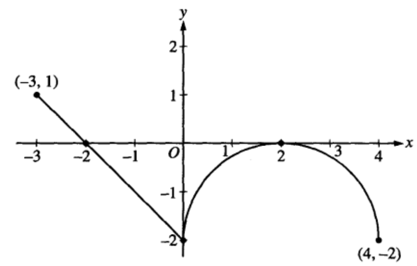
20. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$

21. If f(x) = cos(3x), then f'(π/9) =

22. In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at (3, 2)?

23. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

24. Let f be a function defined on the closed interval $-3 \le x \le 4$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown to the right.



Graph of f'

a) On what intervals, if any, is f increasing?

b) On what intervals, if any, is f concave upward?

c) Find the x–coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$.

d) Find an equation for the line tangent to the graph of f at the point (0, 3).

25. Let f be the function with derivative given by $f'(x) = x^2 - 2/x$. On which of the following intervals is f decreasing?

26. If $y = 3x - 6$, what is the minimum value of the product xy?

~~27. Let g be a twice-differentiable function with g'(x) > 0 and g''(x) > 0 for all real numbers x, such that g(4) = 12 and g(5) = 18. Of the following, which is a possible value for g(6)?~~

28. If $f(x) = x^2 + 2x$, then $\frac{dy}{dx}(f(\ln x)) =$

29. If cos(xy) = x, then dy/dx =

30. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

31. The volume V of a cone ($V = 1/3\ \pi r^2 h$) is increasing at the rate of 28π cubic ft. per second. At the instant when the radius, r, of the cone is 3 ft., its volume is 12π cubic ft. and the radius is increasing at 1/2 ft. per second.

(a) What is the rate of change of the area of its base? (b) What is the rate of change of its height, h?

32. If $y = 5^x + 4x - 2$, Find dy/dx.

33. If $f(x) = \ln(x + 2 + e^x)$, then $f'(0)$ is

34. If $y = \cos(2x)$, find dy/dx.

35. If $y = x^3 \sin(5x)$, find dy/dx.

36. What is the slope of the line tangent to the curve $2y^2 - x^2 = 3 - 3xy$ at the point (3, 2)?

37. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

38. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point $x = \frac{1}{4}$?