Calculus

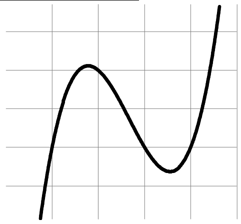
Write your questions and thoughts here!

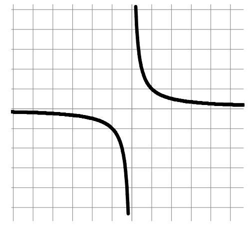
1.4 Continuity

Name:____________________________

Notes

Defining Continuity:





Formal Definition of Continuity:

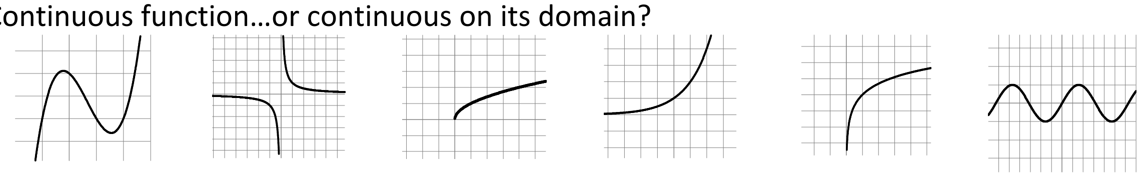
For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1.

2.

3.

Continuous function…or continuous on its domain?



Types of Discontinuities:

1.

2.

3.

For each function identify the x value and type of each discontinuity.

1. $f(x) = \frac{x^2-8x+12}{x^2+3x-10}$	2. $f(x) = \sqrt{2x-3}$	3. $g(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ x + 2, & -1 \le x < 2 \\ 2^x, & x \ge 2 \end{cases}$

Write your questions and thoughts here!

1.4 Continuity

Notes

Finding the Domain

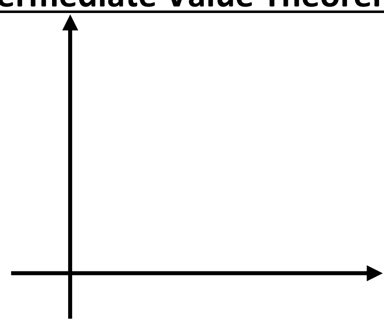
Two scenarios to watch for when looking for a **restriction** on the domain.	
1.	$f(x) = \frac{x-5}{x+1}$
2.	$f(x) = \sqrt{7x+3}$

Find the domain of each function.

4. $f(x) = \frac{3x}{x\sqrt{x+5}}$

5. $h(x) = \frac{5}{2-\sqrt{x}}$

Intermediate Value Theorem (for continuous functions) - IVT



6. Use the IVT to answer the following questions if $f(x) = x^3 - 2x - 5$.
   a. Find $f(1)$.
   b. Find $f(2)$.
   c. Find $f(3)$.
   d. Does the function have a zero? How do you know?

Now summarize what you learned!

1.4 Continuity

Calculus   Name: ______________________________   **Practice**

Identify and classify each point of discontinuity of the given function.

1. $f(x) = \frac{x}{x+1}$

2. $f(x) = \frac{x^2}{x^2+3x}$

3. $f(x) = \frac{2x}{2x-5}$

4. $f(x) = \sqrt{2-6x}$

5. $f(x) = \frac{x+2}{x^2-2x-8}$

6. $f(x) = \frac{4x+5}{3}$

7. $f(x) = \begin{cases} 3-2x, & x < 2 \\ x-3, & x \geq 2 \end{cases}$

8. $f(x) = \begin{cases} 5x+1, & x \leq -1 \\ x+3, & x > -1 \end{cases}$

9. $f(x) = \begin{cases} \frac{x^2-1}{x+11}, & x < 4 \\ x-3, & x > 4 \\ 5, & x = 4 \end{cases}$

10. $f(x) = \begin{cases} \frac{x}{e}+3, & x < e \\ \ln x^4, & x \geq e \end{cases}$

Find the domain of each function.

11. $s(x) = \frac{\sqrt{6x-2}}{5}$

12. $w(t) = \frac{6}{\sqrt{2t+10}}$

13. $f(x) = \frac{x}{\sqrt{6-2x}}$

14. $v(t) = \frac{3t}{t\sqrt{t+7}}$

15. $g(x) = \frac{x+1}{x^2+5x+4}$

16. $g(w) = \frac{2}{4-\sqrt{w}}$

17. $s(t) = \sqrt[3]{t-8}$

18. $h(t) = \frac{\sqrt{4-t}}{t-5}$

19. $g(x) = x^2 + 11x + 30$

Below is a table of values for a continuous function f. Use this table to answer questions 20-22.

x	3	4	5	6	7
$f(x)$	4	1	-3	-1	6

20. On the interval $3 \le x \le 7$, must there be a value of x for which $f(x) = 5$? Explain.

21. On the interval $3 \le x \le 7$, ***could*** there be a value of x for which $f(x) = 7$? Explain.

22. What is the minimum number of zeros f must have on the interval $3 \le x \le 7$?

Below is a table of values for a continuous function g. Use this table to answer questions 23-26.

x	0	2	15	32	50
$g(x)$	-1	10	17	-10	8

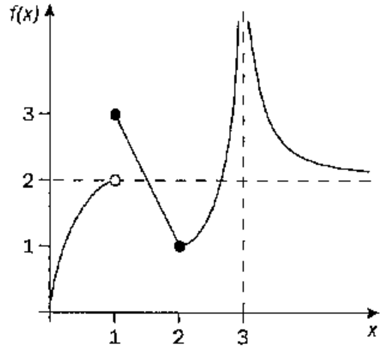
23. On the interval $0 \le x \le 15$, must there be a value of x for which $g(x) = -3$? Explain.

24. On the interval $0 \le x \le 50$, must there be a value of x for which $g(x) = 11$? Explain.

25. What is the minimum number of zeros g must have on the interval $15 \le x \le 50$?

26. What is the minimum number of zeros g must have on the interval $0 \le x \le 50$?

1. The graph of the function $f(x)$ is shown below:



Which of the following statements is true about f?

I. f is undefined at $x = 1$.

II. f is defined but not continuous at $x = 2$.

III. f is defined and continuous at $x = 3$.

(A) Only I (B) Only II (C) I and II (D) I and III (E) None of the statements are true.

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2. Let $y = \frac{x^2+4x-21}{x^2-9}$. This function has a hole. What is the y-value of the hole?

(A) $\frac{5}{3}$ (B) 3 (C) $-\frac{10}{3}$ (D) 0 (E) -3

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3. For which value of k is the following function continuous at $x = 4$?

$$f(x) = \begin{cases} \sin\dfrac{\pi}{x}, & x \leq 4 \\ k\sqrt{\dfrac{x}{2}}, & x > 4 \end{cases}$$

(A) $k = 2$ (B) $k = 1$ (C) $k = -1$ (D) $k = \frac{1}{2}$ (E) $k = -\frac{1}{2}$

4.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $g(x) = \frac{1}{2}$ must have at least two intersections with f in the interval $[0, 2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

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5. For what value of k will the function $f(x) = \frac{x^2-(k+2)x+6}{x-k}$ have a point discontinuity at $x = k$?

(A) $k = -1$ (B) $k = 0$ (C) $k = 1$ (D) $k = 2$ (E) $k = 3$

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6. Suppose f is continuous on the closed interval $[0,4]$ and suppose $f(0) = 1, f(1) = 2, f(2) = 0,$ $f(3) = -3, f(4) = 3$. Which of the following statements about the zeros of f on $[0,4]$ is always true?

(A) f has exactly one zero on $[0, 4]$.

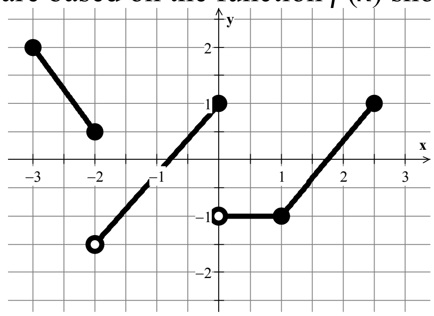
(B) f has more than one zero on $[0, 4]$.

(C) f has more than two zeros on $[0, 4]$.

(D) f has exactly two zeros on $[0, 4]$.

(E) None of the statements above is true.

Questions 7 through 9 are based on the function $f(x)$ shown in the graph below:



7. The function $f(x)$ has a removable discontinuity at:

(A) $x = -2$ only   (B) $x = 0$ only   (C) $x = 1$ only

(D) $x = -2$ and $x = 0$ only   (E) $f(x)$ has no removable discontinuities.

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8. On what intervals is $f(x)$ continuous?

(A) $[-3, -2] \cup [-2, 0] \cup [0, 2.5]$ (B) $[-3, -2] \cup (-2, 0] \cup [0, 2.5]$

(C) $[-3, -2] \cup (-2, 0] \cup (0, 2.5]$ (D) $[-3, -2] \cup [-2, 0] \cup (0, 2.5]$

(E) $[-3, -2] \cup (-2, 0] \cup (0, 1) \cup (1, 2.5]$

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9. The function has a jump discontinuity at:

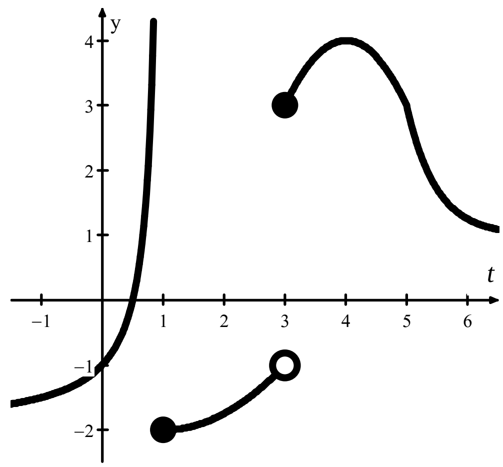
(A) $x = -2$ only   (B) $x = 0$ only   (C) $x = 1$ only

(D) $x = -2$ and $x = 0$ only   (E) $f(x)$ has no jump discontinuities.

FREE RESPONSE ON THE BACK!

For this Free Response problem, answer each question as completely as possible. **Do NOT look at the answers until completed!** When done, use the Solution Key to grade your work. Put your score in the box below.

The graph of a function f is shown below and describes the position of a particle as it moves along the y-axis with respect to time.



a. Describe the movement of the particle on the interval $[1,3]$.

b. Assume $f(t) > 1$ for $t > 6$, and $y = 1$ is an asymptote. Describe the movement of the particle as t approaches infinity.

c. Can we use the Intermediate Value Theorem on the interval $[-1,2]$ to show that f has a zero in that interval? On the interval $[2,5]$? Explain your reasoning.