10.4 Separation of Variables

Solve the differential equation.

\[ \frac{dy}{dx} = \frac{x^2}{y} \]

\[ \frac{dy}{dx} = (\sin x)y^2 \]

**Initial Value**

Solve for \( y \) if \( \frac{dy}{dx} = (xy)^2 \) and \( y = 1 \) when \( x = 1 \)
Use the differential to answer the following:

\[ \frac{dy}{dx} = \frac{2x}{y} \]

(a) Fill in the slope field

(b) Write the equation of the line tangent to the solution curve at point (2,1).

(c) Find the particular solution with initial condition of \( f(2) = 1 \).

Solve the differential equation.

\[ \frac{dy}{dx} = (y + 2)e^x \]

(a) Sketch a particular solution through the point (0, -1)

(b) Find the particular solution with initial condition (0, -1)

SUMMARY:
### Solve the differential equation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \frac{dy}{dx} = \frac{3x^2}{y} )</td>
</tr>
<tr>
<td>2.</td>
<td>( \frac{dy}{dx} = e^x y^2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( \frac{dy}{dx} = -2x(y - 3) )</td>
</tr>
<tr>
<td>4.</td>
<td>( \frac{dy}{dx} = x \cos x^2 )</td>
</tr>
</tbody>
</table>

### Find the solution that satisfies the given condition.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>( \frac{dy}{dx} = y \sin x ) if ( y(0) = 2 )</td>
</tr>
<tr>
<td>6.</td>
<td>( \frac{dy}{dx} = \frac{e^x}{y} ) if ( y(0) = -4 )</td>
</tr>
</tbody>
</table>
Find the solution that satisfies the given condition.

7. \( \frac{dy}{dx} = xy^2 \) and \( y = 1 \) when \( x = 0 \)

8. \( \frac{dy}{dx} = \frac{1}{5}(8 - y) \) and \( y = 6 \) when \( x = 0 \)

Use the differential equation and its slope field to answer the following.

9. \( \frac{dy}{dx} = (y + 5)(x + 2) \)
   
a. Sketch a particular solution through the point \((0,1)\).
   
b. Find the particular solution \( y = f(x) \) when \( f(0) = 1 \)

10. \( \frac{dy}{dx} = e^{x-y} \)
    
a. Sketch a particular solution through the point \((0,2)\).
    
b. Find the particular solution \( y = f(x) \) when \( f(0) = 2 \)
10.4 Separation of Variables

MULTIPLE CHOICE

1. \[ \int_{-1}^{1} \frac{4}{1 + x^2} \, dx = \]

(A) 0
(B) \( \pi \)
(C) 1
(D) 2\( \pi \)
(E) 2

2. If \( \frac{dy}{dx} = \frac{(3x^2 + 2)}{y} \) and \( y = 4 \) when \( x = 2 \), then when \( x = 3 \), \( y = \)

(A) 18
(B) \( \pm \sqrt{66} \)
(C) 58
(D) \( \pm \sqrt{74} \)
(E) \( \pm \sqrt{58} \)

3. If \( \frac{dy}{dx} = \frac{x^3 + 1}{y} \) and \( y = 2 \) when \( x = 1 \), then when \( x = 2 \), \( y = \)

(A) \( \sqrt{\frac{27}{2}} \)
(B) \( \sqrt{\frac{27}{8}} \)
(C) \( \pm \sqrt{\frac{27}{8}} \)
(D) \( \pm \frac{3}{2} \)
(E) \( \pm \sqrt{\frac{27}{2}} \)

4. If \( \frac{dy}{dt} = -2y \) and if \( y = 1 \) when \( t = 0 \), what is the value of \( t \) for which \( y = \frac{1}{2} \)?

(A) \( -\frac{1}{2} \ln 2 \)
(B) \( -\frac{1}{4} \)
(C) \( \frac{1}{2} \ln 2 \)
(D) \( \sqrt{2} \)
(E) \( \ln 2 \)
5. What is the equation of the line tangent to the graph \( y = \sin^2 x \) at \( x = \frac{\pi}{4} \)?

(A) \( y - \frac{1}{2} = -\left( x - \frac{\pi}{4} \right) \)

(B) \( y - \frac{1}{2} = \left( x - \frac{\pi}{4} \right) \)

(C) \( y - \frac{1}{\sqrt{2}} = \left( x - \frac{\pi}{4} \right) \)

(D) \( y - \frac{1}{\sqrt{2}} = \frac{1}{2} \left( x - \frac{\pi}{4} \right) \)

(E) \( y - \frac{1}{2} = \frac{1}{2} \left( x - \frac{\pi}{4} \right) \)

---

FREE RESPONSE

YOUR SCORE:____ out of 9

6. Consider the differential equation \( \frac{dy}{dx} = e^{y(3x^2 - 6x)} \). Let \( y = f(x) \) be the particular solution to the differential equation that passes through (1,0).

(a) Write an equation for the line tangent to the graph of \( f \) at the point (1,0). Use the tangent line to approximate \( f(1.2) \).

(b) Find \( y = f(x) \), the particular solution to the differential equation that passes through (1,0).