Calculus

11.1 Area Between Curves

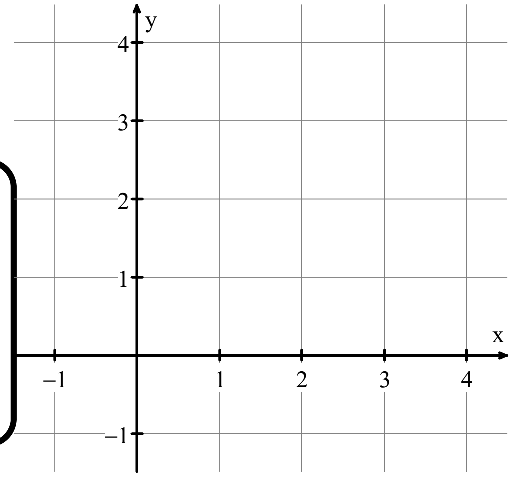
Name:________________________

Write your questions and thoughts here!

Notes

Recall:

- Find the area under the curve of $f(x)$



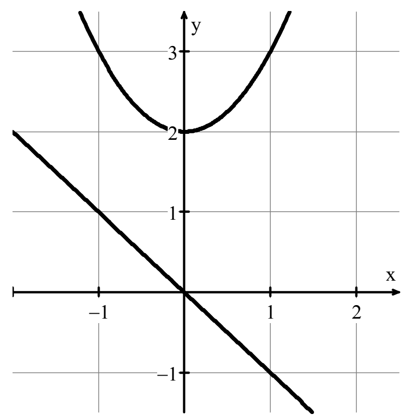
How would you find the area between two curves?

Area Between Two Curves:

$$A = \int_a^b [\qquad\qquad\qquad] \, dx$$

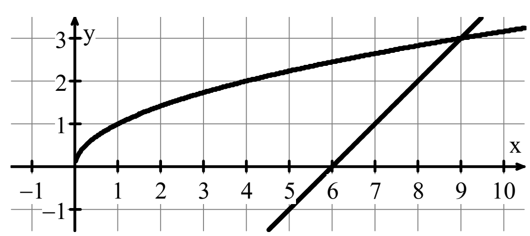
$\geq$ for all x in $[a, b]$

1. Find the area bounded by the curves of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.



2. Find the area bounded by $y = 2 - x^2$ and $y = x$.

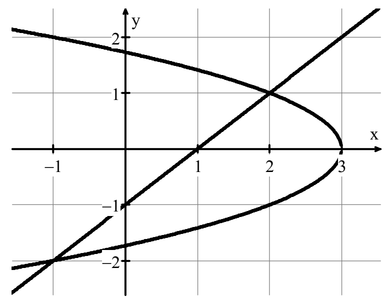
3. Set up the integral that allows you to find the area in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by $y = x - 6$.



Write your questions and thoughts here!

4. Set up the integrals to find the area bounded by $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

5. Set up the integral to find the area bounded by $x = 3 - y^2$ and $x = y + 1$.



Now summarize what you learned!

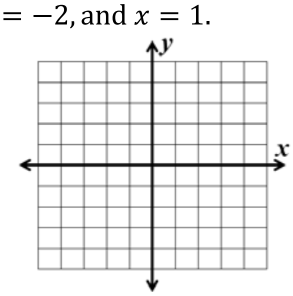
11.1 Area Between Curves

Calculus

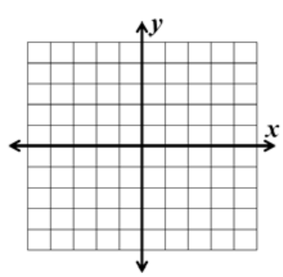
Practice

Sketch **the graph of each equation, then set up the integral to find the area of the region bounded by the graphs. Do NOT evaluate, just set up the integral!**

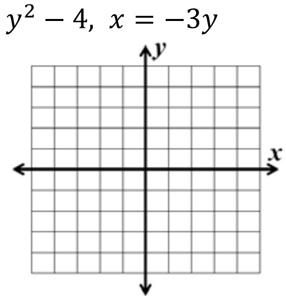
1. $f(x) = x^2 + 2$, $g(x) = -x$, $x = -2$, and $x = 1$.



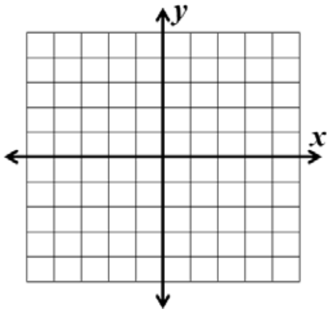
2. $f(x) = 6 - x^2$ and $g(x) = x$



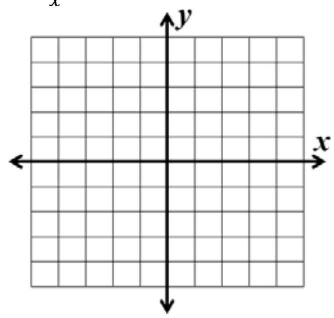
3. $x = y^2 - 4$, $x = -3y$



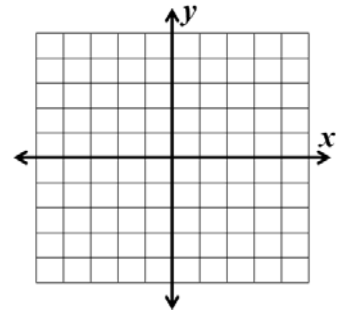
4. $y = x, \ y = 2 - x, \ y = 0$



5. $y = \frac{1}{x^2}, \ y = 0, \ x = 1, \ x = 5$



6. $x = y^2, \ x = y + 2$



7. $f(x) = 2x^3 - x^2 - 7x, \ g(x) = x^2 + 5x$

Find the area of the region bounded by the given equations. Evaluate an integral with respect to *x* (perpendicular to the *x*-axis) by using a calculator. Find the same area by evaluating an integral with respect to *y* (perpendicular to the *y*-axis) by showing your work.

8. $y = x^2$ and $y = x^3$ *Sketch* your graph here in the middle!

with respect to *x* (and a calculator) with respect to *y* (show work)

9. $y = \sqrt{x}, x = 0$ and $y = x - 2$ *Sketch* your graph here in the middle!

with respect to *x* (and a calculator) with respect to *y* (show work)

10. $y = x^2, y = -x, x = 0, x = 2$ *Sketch* your graph here in the middle!

with respect to x (and a calculator) ⇩ with respect to y (show work)

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11. $x = \frac{1}{2}y^2, y = 1, y = -\frac{1}{2}x + 3$ *Sketch* your graph here in the middle!

with respect to x (and a calculator) ⇩ with respect to y (show work)

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12. $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$ *Sketch* your graph here in the middle!

with respect to x (and a calculator)
You need a graphing calculator to help you find the boundaries.

⇩

with respect to y (USE A CALCULATOR)

11.1 Area Between Curves

Test Prep

1. Which of the following functions grows the fastest?

(A) $a(u) = \left(\frac{1}{2}\right)^u$ (B) $b(u) = u^{100} + u^{99}$ (C) $c(u) = 4^u$

(D) $d(u) = 200e^u$ (E) $e(u) = e^u + u^3$

2. If $0 \leq k \leq \frac{\pi}{2}$ and the area under the curve $y = \sin x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.75, then $k =$

(A) 1.318 (B) 0.848 (C) 0.723 (D) 0.533 (E) 0.253

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3. Let $f(x) = 5x \sec x + x^3 \cos x + 17\pi$. Determine $\frac{d}{dx} f(x)$.

(A) $5 \sec x \tan x + 3x^2 \cos x + 17\pi$

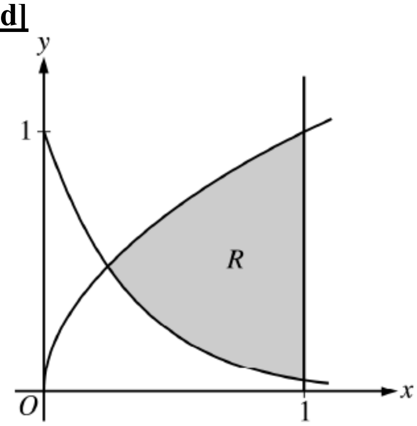
(B) $5 \sec^2 x - x^3 \sin x$

(C) $5 \sec x \tan x - 3x^2 \sin x$

(D) $5 \sec x + 5x \sec x \tan x + 3x^2 \cos x - x^3 \sin x$

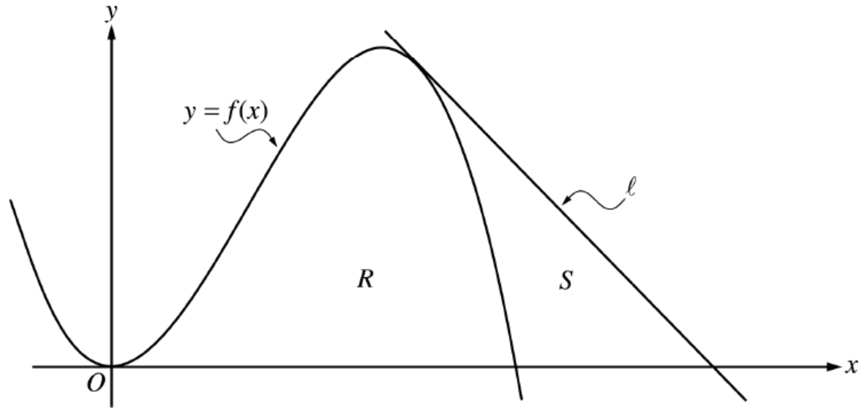
(E) $5 \sec x + 5x \sec x \tan x - 3x^2 \cos x + x^3 \sin x + 17\pi$

2003 Form A #1 [calculator allowed]



Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above. Find the area R.

2003 Form B #1 [calculator allowed]



Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.