11.3 Solids of Revolution (Washers)

Calculus

Solutions

Practice

Find the volume of the solid formed by revolving the region about the x-axis.

1. $y = x^2,\ y = x^3$

$V = \pi\int_0^1 (x^2)^2 - (x^3)^2\,dx$

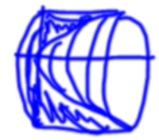
$\pi\int_0^1 x^4 - x^6\,dx$

$\pi\left[\frac{x^5}{5} - \frac{x^7}{7}\right]\Big|_0^1$

$\pi\left(\frac{1}{5} - \frac{1}{7}\right)$

$\boxed{\frac{2}{35}\pi}$

2. $y = \sqrt{x},\ x = 0,\ y = 2$



$V = \pi\int_0^4 (2)^2 - (\sqrt{x})^2\,dx$

$\pi\int_0^4 4 - x\,dx$

$\pi\left[4x - \frac{x^2}{2}\right]\Big|_0^4$

$\pi(16 - 8) = \boxed{8\pi}$

Find the volume of the solid formed by revolving the region about the y-axis.

3. $y = x^2,\ y = x^3$

$x = \pm\sqrt{y} \quad x = \sqrt[3]{y}$

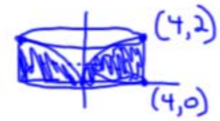
$\pi\int_0^1 (\sqrt[3]{y})^2 - (\sqrt{y})^2\,dy$

$\pi\int_0^1 y^{2/3} - y\,dy$

$\pi\left[\frac{3}{5}y^{5/3} - \frac{1}{2}y^2\right]\Big|_0^1$

$\pi\left(\frac{3}{5} - \frac{1}{2}\right) = \boxed{\frac{1}{10}\pi}$

4. $y = \sqrt{x},\ y = 0,\ x = 4$



$y^2 = x$

$\pi\int_0^2 4^2 - (y^2)^2\,dy$

$\pi\int_0^2 16 - y^4\,dy$

$\pi\left[16y - \frac{y^5}{5}\right]\Big|_0^2$

$\pi\left(32 - \frac{32}{5}\right) = \boxed{\frac{128}{5}\pi}$

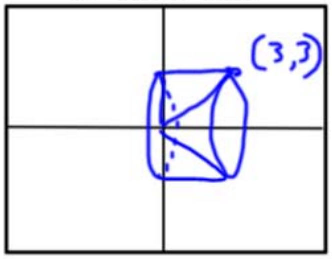
5. Sketch the graph and find the area of the region bounded by $y = x$, $x = 0$, and $y = 3$

$\int_0^3 (3-x)\,dx$

$3x - \frac{x^2}{2}\Big|_0^3 = 9 - \frac{9}{2} = \boxed{\frac{9}{2}}$

Set up the integral to find the volume when revolving it about the given line. DO NOT EVALUATE!

a. The x-axis.

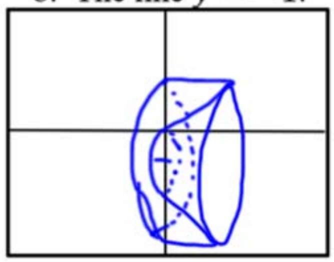


$R = 3$

$r = x$

$V = \pi\int_0^3 9 - x^2\,dx$

b. The line $y = -1$.

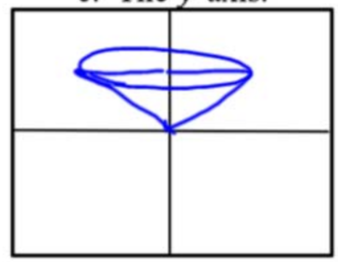


$R = 4$

$r = x+1$

$V = \pi\int_0^3 16 - (x+1)^2\,dx$

c. The y-axis.

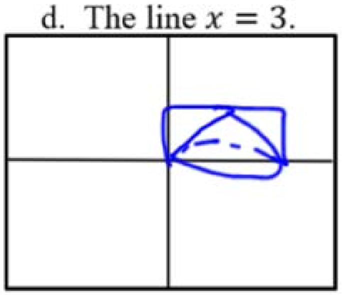


$R = y$

$r = 0$

$V = \pi\int_0^3 y^2\,dy$

d. The line $x = 3$.

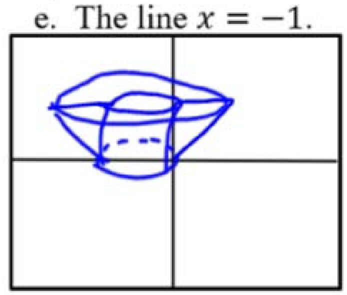


$R = 3$

$r = 3 - y$

$V = \pi\int_0^3 9 - (3-y)^2\,dy$

e. The line $x = -1$.



$R = y + 1$

$r = 1$

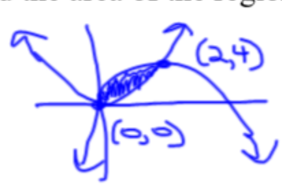
$V = \pi\int_0^3 (y+1)^2 - 1\,dy$

6. Sketch the graph and find the area of the region bounded by $y = x^2$ and $y = 4x - x^2$.

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0 \quad x = 2$



$A = \int_0^2 4x - x^2 - x^2\,dx$

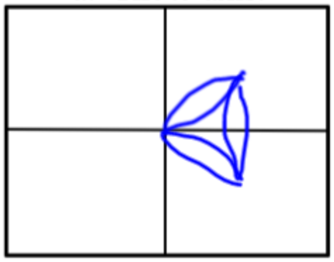
$\int_0^2 4x - 2x^2\,dx$

$2x^2 - \frac{2}{3}x^3 \Big|_0^2$

$8 - \frac{16}{3} = \boxed{\frac{8}{3}}$

Set up the integral to find the volume when revolving it about the given line.
$y = x^2$ and $y = 4x - x^2$. DO NOT EVALUATE!

a. The x-axis.

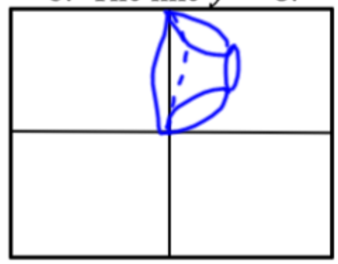


$R = 4x - x^2$

$r = x^2$

$V = \pi \int_0^2 (4x - x^2)^2 - x^4 \, dx$

b. The line $y = 6$.



$R = 6 - x^2$ or $x^2 - 6$

$r = 6 - 4x + x^2$ or $4x - x^2 - 6$

$V = \pi \int_0^2 (6 - x^2)^2 - (6 - 4x + x^2)^2 \, dx$

7. Sketch the graph and find the area of the region bounded by $y = x^2$, and $y = \sqrt[3]{x}$

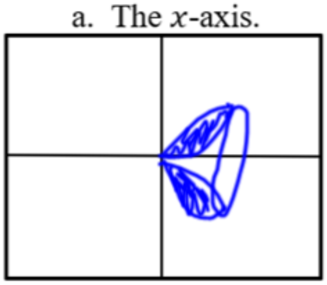
$A = \int_0^1 x^{1/3} - x^2 \, dx$

$\frac{3}{4} x^{4/3} - \frac{1}{3} x^3 \Big|_0^1$

$\frac{3}{4} - \frac{1}{3} \longrightarrow \boxed{\frac{5}{12}}$

Set up the integral to find the volume when revolving it about the given line. DO NOT EVALUATE!

a. The x-axis.

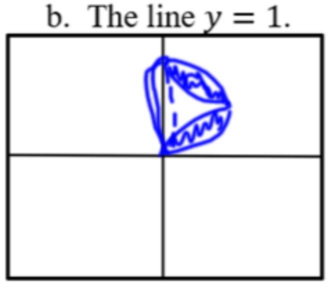


$R = \sqrt[3]{x}$

$r = x^2$

$V = \pi \int_0^1 x^{2/3} - x^4 \, dx$

b. The line $y = 1$.

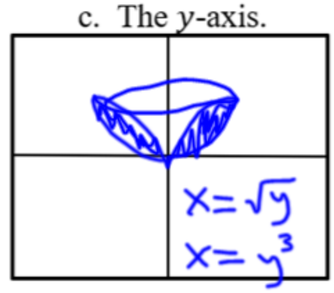


$R = 1 - x^2$

$r = 1 - \sqrt[3]{x}$

$V = \pi \int_0^1 (1 - x^2)^2 - (1 - \sqrt[3]{x})^2 \, dx$

c. The y-axis.

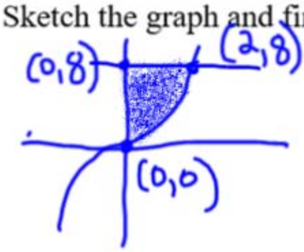


$R = \sqrt{y}$

$r = y^3$

$V = \pi \int_0^1 y - y^6 \, dy$

8. Sketch the graph and find the area of the region bounded by $y = x^3$, $x = 0$, and $y = 8$.



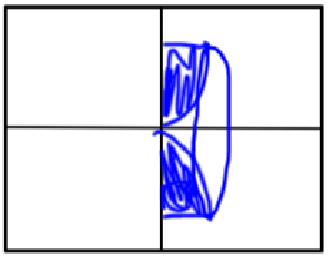
$A = \int_0^2 8 - x^3 \, dx$

$8x - \frac{1}{4} x^4 \Big|_0^2$

$16 - 4 \longrightarrow \boxed{12}$

Set up the integral to find the volume when revolving it about the given line.
$y = x^3$, $x = 0$, and $y = 8$. DO NOT EVALUATE!

a. The x-axis.

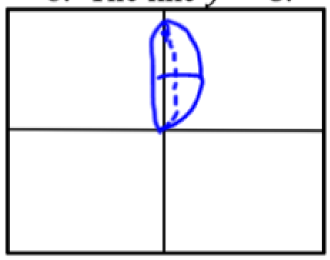


$R = 8$

$r = x^3$

$V = \pi\int_0^2 64 - x^6 \, dx$

b. The line $y = 8$.

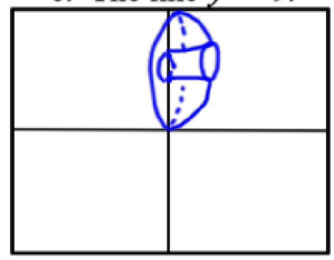


$R = 8 - x^3$ or $x^3 - 8$

$r = 0$

$V = \pi\int_0^2 (8 - x^3)^2 \, dx$

c. The line $y = 9$.

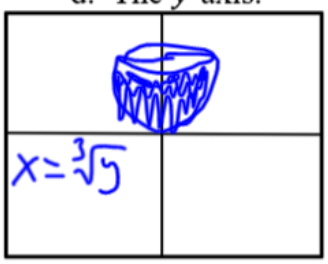


$R = 9 - x^3$ or $x^3 - 9$

$r = 1$

$V = \pi\int_0^2 (9 - x^3)^2 - 1 \, dx$

d. The y-axis.

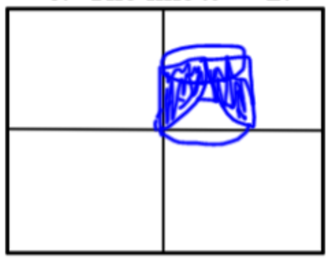


$R = \sqrt[3]{y}$

$r = 0$

$V = \pi\int_0^8 y^{2/3} \, dy$

e. The line $x = 2$.

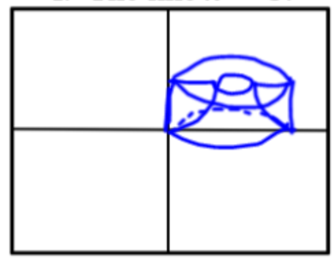


$R = 2$

$r = 2 - \sqrt[3]{y}$

$V = \pi\int_0^8 4 - (2 - \sqrt[3]{y})^2 \, dy$

f. The line $x = 3$.



$R = 3$

$r = 3 - \sqrt[3]{y}$

$V = \pi\int_0^8 9 - (3 - \sqrt[3]{y})^2 \, dy$

Test Prep: 1C

2003 Form A #

(b) Volume $= \pi\int_T^1 \left((1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \right) dx$

$= 0.453\pi$ or 1.423 or 1.424

3 :
- 2 : integrand
  - $\langle -1 \rangle$ reversal
  - $\langle -1 \rangle$ error with constant
  - $\langle -1 \rangle$ omits 1 in one radius
  - $\langle -2 \rangle$ other errors
- 1 : answer

2003 Form B #1

(c) Volume $= \pi\int_0^4 (4x^2 - x^3)^2 \, dx$

$= 156.038\pi$ or 490.208

3 :
- 1 : limits and constant
- 1 : integrand
- 1 : answer