PRACTICE

Find the derivative of the following.	
1. $f(x) = (3x^2 - 1)^5$ $f'(x) = 5(3x^2-1)^4(6x)$ $f'(x) = 30x(3x^2-1)^4$	2. $f(r) = \sqrt[3]{5r^2 - 2r + 1}$ $f'(r) = \frac{10r-2}{3\sqrt[3]{(5r^2-2r+1)^2}}$
3. $y = \frac{1}{(7x^2-1)^2}$ $y = (7x^2-1)^{-2}$ $y' = -2(7x^2-1)^{-3}(14x)$ $y' = \frac{-28x}{(7x^2-1)^3}$	4. $h(x) = 2\sqrt{3x^2 - 5}$ $h'(x) = \frac{6x}{\sqrt{3x^2-5}}$
5. $f(x) = (\pi x - 1)^2 + 7$ $f'(x) = 2(\pi x - 1)^1 \cdot \pi$ $f'(x) = 2\pi(\pi x - 1)$	6. $g(x) = 4x - \frac{3}{\sqrt{2x+1}}$ $g'(x) = 4 + \frac{3}{\sqrt{(2x+1)^3}}$

Find the derivatives of the following.	
7. $y = x\sqrt{2x-1}$ $u \cdot v$ $u = x \quad v = (2x-1)^{1/2}$ $u' = 1 \quad v' = \frac{1}{2}(2x-1)^{-1/2}(2)$ $= \frac{1}{\sqrt{2x-1}}$ $u'v + uv'$ $(1)\sqrt{2x-1} + (x)\frac{1}{\sqrt{2x-1}}$ $\sqrt{2x-1} + \frac{x}{\sqrt{2x-1}}$	8. $y = (x^3 + e)^{-2}$ $y' = \frac{-6x^2}{(x^3+e)^3}$
9. $g(x) = 2x(x^3 - 1)^2$ $u \cdot v$ $u = 2x \quad v = (x^3-1)^2$ $u' = 2 \quad v' = 2(x^3-1)^1(3x^2)$ $= 6x^2(x^3-1)$ $= 6x^5 - 6x^2$ $u'v + uv'$ $2(x^3-1)^2 + 2x(6x^5-6x^2)$ $2(x^3-1)^2 + 12x^6 - 12x^3$	10. $h(x) = \frac{6x^2-5}{\sqrt{2-5x}}$ $h'(x) = \frac{12x\sqrt{2-5x} + \frac{5(6x^2-5)}{2\sqrt{2-5x}}}{2-5x}$

Evaluate the derivative at a point.

11. $f(x) = \sqrt{1 + (x^2 - 1)^3}$

$f'(2) =$

$f(x) = \left(1 + (x^2-1)^3\right)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}\left(1 + (x^2-1)^3\right)^{-1/2} \cdot 3(x^2-1)^2 \cdot \cancel{2}x$

$f'(x) = \frac{3x(x^2-1)^2}{\sqrt{1+(x^2-1)^3}}$

$f'(2) = \frac{3(2)(2^2-1)^2}{\sqrt{1+(2^2-1)^3}} = \frac{54}{\sqrt{28}}$

simplifies to $\frac{27\sqrt{7}}{7}$

12. $y = \frac{x+1}{\sqrt{2x-1}}$

$\left.\frac{dy}{dx}\right|_{x=1}$

-1

Write the equation of the tangent line and the normal line at the point given.

13. $f(x) = \sqrt{x^2 - 9}$ at $x = 5$

$f(5) = \sqrt{5^2 - 9}$

$\boxed{f(5) = 4}$

$f(x) = (x^2-9)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(x^2-9)^{-1/2}(\cancel{2}x)$

$f'(x) = \frac{x}{\sqrt{x^2-9}}$

$f'(5) = \frac{5}{\sqrt{5^2-9}} = \frac{5}{\sqrt{16}} = \frac{5}{4}$

Tangent Line

$y - 4 = \frac{5}{4}(x-5)$

Normal Line

$y - 4 = -\frac{4}{5}(x-5)$

14. $f(x) = \frac{1}{(3-2x)^2}$ at $x = 1$

Tangent Line

$y - 1 = 4(x-1)$

Normal Line

$y - 1 = -\frac{1}{4}(x-1)$

Particle Motion

15. The position of a particle moving along a coordinate line is $s = \sqrt{1+4t}$, with s in meters and t in seconds. Find the particle's velocity at $t = 6$.

$s = (1+4t)^{1/2}$

$v(t) = s'(t) = \frac{1}{\cancel{2}}(1+4t)^{-1/2}(\cancel{4})$

$s'(t) = \frac{2}{\sqrt{1+4t}}$

$s'(6) = \frac{2}{\sqrt{1+4(6)}} = \frac{2}{\sqrt{25}} = \frac{2}{5}$ meters per second

16. If $s = \frac{t}{t^2+5}$ is the position function of a moving particle for $t \geq 0$, then at what instant of time will the particle start to reverse its direction of motion and where is it at the instant?

$s' = \frac{-t^2+5}{(t^2+5)^2}$

$s'(t) = 0$

$t = \sqrt{5}$ seconds

So at $\sqrt{5}$ seconds the particle has no velocity. Did it reverse?

Approaching $\sqrt{5}$ from the left

$s'(0) = \frac{5}{25}$ positive

Approaching $\sqrt{5}$ from right

$s'(3) = \frac{-4}{14^2}$ negative

Change from positive to negative! so it reversed direction

Position:

$S(\sqrt{5}) = \frac{\sqrt{5}}{5+5} = \boxed{\frac{\sqrt{5}}{10}}$

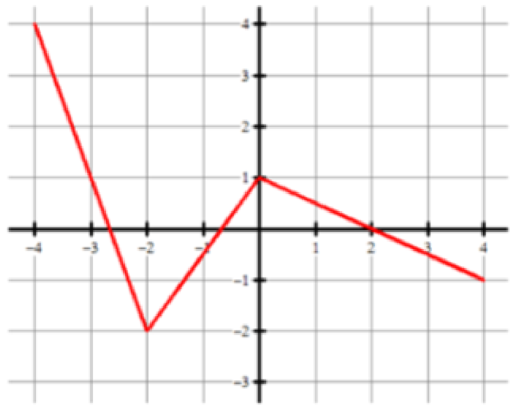
Find $f'(5)$ given the following.		
	17. $f(x) = g(x) + h(x)$ $f'(x) = g'(x) + h'(x)$ $f'(5) = g'(5) + h'(5)$ $f'(5) = 6 + -4 = 2$	18. $f(x) = (h(x))^2$ -40
$g(5) = 9$ and $g'(5) = 6$ $h(5) = 5$ and $h'(5) = -4$	19. $f(x) = \sqrt{g(x)}$ $f(x) = [g(x)]^{1/2}$ $f'(x) = \frac{1}{2}[g(x)]^{-1/2} g'(x)$ $f'(5) = \frac{g'(5)}{2\sqrt{g(5)}} = \frac{6}{2\sqrt{9}} = \frac{6}{6} = 1$	20. $f(x) = 2g(x)h(x)$ -12
	21. $f(x) = \frac{1}{h(x)}$ $f(x) = (h(x))^{-1}$ $f'(x) = -1(h(x))^{-2} h'(x)$ $f'(5) = \frac{-h'(5)}{(h(5))^2} = \frac{--4}{5^2} = \frac{4}{25}$	22. $f(x) = g(h(x))$ -24

TEST PREP

1. **B**
2. **B**
3. **B**
4. **D**
5. **B**

FREE RESPONSE **Your score: ____ out of 4**

1. The graph of the function f, shown below, consists of three line segments. Suppose $g(x)$ is a function whose derivative is f.



Graph of f

(a) Suppose $y = x + 7$ is the equation for the line tangent to the graph of $g(x)$ at $x = -3$. Let h be the function defined by $h(x) = (g(x))^2$. Find $h'(-3)$.

$h'(3) = 2\big(g(-3)\big)g'(-3)$ ⟵ 1 point for correct derivative set up

$h'(3) = 2(4)(1) = 8$ ⟵ 1 point for the solution

(b) Describe the shape of the graph of $g(x)$ near $x = 2$.

As x approaches 2 from the left, the derivative is positive meaning the function is increasing.
As x approaches 2 from the right, the derivative is negative meaning the function is decreasing.
At $x = 2$, the derivative is zero which means the slope of the tangent line is zero causing a maximum or minimum point. Since the function is increasing and then decreasing it must be a maximum point.

1 point for correct explanation

(c) Give a piecewise defined equation for $g''(x)$.

$$f(x) = \begin{cases} -3 & -4 < x < -2 \\ \frac{3}{2} & -2 < x < 0 \\ -\frac{1}{2} & 0 < x < 4 \end{cases}$$

1 point for correct function