

4.2 Inverse Derivatives

Name: _____

Write your questions
and thoughts here!

Notes

Recall: Given $f(x) = 5(x - 2)^3 - 1$, find $f^{-1}(x)$.

CONFUSING NOTATION:	<u>Reciprocal</u>		<u>Inverse</u>
	$x^{-1} =$	or	$f^{-1}(x)$ means

Quote from the AP Exam:

“Notation: The inverse of a trigonometric function x may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).”

Inverse Trig Derivatives:

$$\frac{d}{dx} \sin^{-1}(x) =$$

$$\frac{d}{dx} \cos^{-1}(x) =$$

$$\frac{d}{dx} \sec^{-1}(x) =$$

$$\frac{d}{dx} \csc^{-1}(x) =$$

$$\frac{d}{dx} \tan^{-1}(x) =$$

$$\frac{d}{dx} \cot^{-1}(x) =$$

Find the derivative.

1. $\frac{d}{dx} \sin^{-1}(3x)$

2. $\frac{d}{dx} \tan^{-1}(2x^2)$

3. $\frac{d}{dx} \sec^{-1}(5x^6)$

4. What is the anti-derivative of $\frac{12x}{\sqrt{1-36x^4}}$?**Derivative of an Inverse Function:**

$$\frac{d}{dx} [f^{-1}(x)] =$$

5. If $f(x) = \cos x - 6x$ and $f^{-1}(-9\pi) = \frac{3\pi}{2}$, find the derivative of $f^{-1}(x)$ at $x = -9\pi$.6. If $f(x) = x^2$, find the derivative of $f^{-1}(x)$ at $x = 4$.

4.2 Inverse Derivatives

Calculus

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Practice

Find the following.

1. $\frac{d}{dx} \sin^{-1}(5x)$	2. $\frac{d}{dx} \csc^{-1}(4x^5)$	3. $\frac{d}{dx} \tan^{-1}(2x)$
4. $\frac{d}{dx} \frac{\sin x}{x}$	5. $\frac{d}{dx} \sec^{-1}(x^3)$	6. $\frac{d}{dx} \csc 6x$
7. $\lim_{x \rightarrow 2} \frac{x-2}{x^2+5x-14}$	8. $\frac{d}{dx} \cos^{-1}(3x^2)$	9. Anti-derivative of $f'(x) = \frac{5}{\sqrt{1-25x^2}}$
10. $\frac{d}{dx} \cot^{-1}(-x)$	11. Anti-derivative of $f'(x) = -\frac{6x^2}{1+4x^6}$	12. $\frac{d}{dx} \log_5 4x$
13. $\frac{d}{dx} \cos^{-1}(-7x)$	14. $\frac{d}{dx} \csc^{-1}(x^7)$	15. $\frac{d}{dx} \cot^{-1}(4x^4)$
16. $\frac{d}{dx} e^{2x^5}$	17. $\frac{d}{dx} \tan^{-1}(\sqrt{x})$	18. $\frac{d}{dx} 5x \sin^{-1}(2x^2)$

19. Anti-derivative of
 $f'(x) = \frac{7}{|x|\sqrt{9x^{14}-1}}$

20. $\frac{d}{dx} \tan(e^x)$

21. $\frac{d}{dx} \sec^{-1}(3 \ln x)$

22. $\frac{d}{dx} \sin^{-1}(\sin x)$

23. $\frac{d}{dx} \frac{15x^3+3x^2+55x}{3x}$

24. Anti-derivative of
 $f'(x) = -\frac{8x}{\sqrt{1-16x^4}}$

INVERSE FUNCTIONS:

25. If $f(x) = 3x^2$ and $f^{-1}(27) = 3$, find the derivative of $f^{-1}(x)$ at $x = 27$

26. If $f(x) = \cos 3x$ and $f^{-1}(0) = \frac{\pi}{6}$, find the derivative of $f^{-1}(x)$ at $x = 0$

27. If $f(x) = x^2 + x$ and $f^{-1}(2) = -2$, find the derivative of $f^{-1}(x)$ at $x = 2$

28. If $f(x) = 6x - 2$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 16$

29. If $f(x) = \frac{\sqrt{x}}{3}$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 1$

30. If $f(x) = 3x^3 - 4$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 20$

4.2 Inverse Derivatives

1. Compute the derivative of $f(x) = \ln x - \sin x + \arctan x + 2^x, x > 0$.

(A) $f'(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + x2^x$

(B) $f'(x) = \frac{1}{x} - \cos x + \frac{1}{1-x^2} + x2^x$

(C) $f'(x) = \frac{1}{x} + \cos x + \frac{1}{1-x^2} + (\ln 2)2^x$

(D) $f'(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

(E) $f'(x) = \frac{1}{x} + \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

2. What is an equation for the line tangent to $y = \tan^{-1} x$ at $x = \sqrt{3}$?

(A) $y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$

(B) $y - \frac{\pi}{6} = -\frac{1}{4}(x - \sqrt{3})$

(C) $y - \frac{\pi}{3} = -\frac{1}{4}(x - \sqrt{3})$

(D) $y - \frac{\pi}{6} = \frac{3}{4}(x - \sqrt{3})$

(E) $y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$

3. $\lim_{x \rightarrow -\infty} \frac{2x+3}{\sqrt{x^2+x+1}}$ is

(A) -2

(B) -1

(C) 0

(D) 2

(E) nonexistent

4. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

- (A) 59 (B) $\frac{1}{59}$ (C) $\frac{1}{10}$ (D) 11 (E) $\frac{1}{11}$
-

5. If $f(x) = x^{\frac{5}{2}}$, then $f'(4)$?

- (A) -10 (B) 24 (C) 5 (D) 10 (E) 20
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FREE RESPONSE

Your score: _____ out of 3

2007 Form A AB3

Use the space below the problem to show work and solutions. Score your answers when completed.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

1. The functions f and g are differentiable for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x .

- (a) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.