

4.2 Inverse Derivatives

Calculus

Name: Solutions

Practice

Find the following.		
<p>1. $\frac{d}{dx} \sin^{-1}(5x)$</p> $\frac{1}{\sqrt{1-(5x)^2}} \cdot 5$ $\frac{5}{\sqrt{1-25x^2}}$	<p>2. $\frac{d}{dx} \csc^{-1}(4x^5)$</p> $-\frac{5}{ x \sqrt{16x^{10}-1}}$	<p>3. $\frac{d}{dx} \tan^{-1}(2x)$</p> $\frac{1}{(2x)^2+1} \cdot 2$ $\frac{2}{4x^2+1}$
<p>4. $\frac{d}{dx} \frac{\sin x}{x}$</p> $\frac{x \cos x - \sin x}{x^2}$	<p>5. $\frac{d}{dx} \sec^{-1}(x^3)$</p> $\frac{1}{ x \sqrt{x^6-1}} \cdot 3x^2$ <p>Pos. Pos.</p> $\frac{3}{ x \sqrt{x^6-1}}$	<p>6. $\frac{d}{dx} \csc 6x$</p> $-6 \csc(6x) \cot(6x)$
<p>7. $\lim_{x \rightarrow 2} \frac{x-2}{x^2+5x-14}$</p> $\frac{x-2}{(x-2)(x+7)}$ $\lim_{x \rightarrow 2} \frac{1}{x+7}$ $\frac{1}{2+7} = \frac{1}{9}$	<p>8. $\frac{d}{dx} \cos^{-1}(3x^2)$</p> $-\frac{6x}{\sqrt{1-9x^4}}$	<p>9. Anti-derivative of $f'(x) = \frac{5}{\sqrt{1-25x^2}}$</p> $\sin^{-1}(5x) + C$
<p>10. $\frac{d}{dx} \cot^{-1}(-x)$</p> $\frac{1}{x^2+1}$	<p>11. Anti-derivative of $f'(x) = -\frac{6x^2}{1+4x^6}$</p> $\cot^{-1}(2x^3) + C$	<p>12. $\frac{d}{dx} \log_5 4x$</p> $\frac{1}{x \ln 5}$
<p>13. $\frac{d}{dx} \cos^{-1}(-7x)$</p> $-\frac{1}{\sqrt{1-(-7x)^2}} \cdot (-7)$ $\frac{7}{\sqrt{1-49x^2}}$	<p>14. $\frac{d}{dx} \csc^{-1}(x^7)$</p> $-\frac{7}{ x \sqrt{x^{14}-1}}$	<p>15. $\frac{d}{dx} \cot^{-1}(4x^4)$</p> $-\frac{1}{(4x^4)^2+1} \cdot 16x^3$ $-\frac{16x^3}{16x^8+1}$
<p>16. $\frac{d}{dx} e^{2x^5}$</p> $10x^4 e^{2x^5}$	<p>17. $\frac{d}{dx} \tan^{-1}(\sqrt{x})$</p> $\frac{1}{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}}$ $\frac{1}{2\sqrt{x}(x+1)}$	<p>18. $\frac{d}{dx} 5x \sin^{-1}(2x^2)$</p> $5 \sin^{-1}(2x^2) + \frac{20x^2}{\sqrt{1-4x^4}}$

19. Anti-derivative of
 $f'(x) = \frac{7}{|x|\sqrt{9x^{14}-1}}$
 $\sqrt{9x^{14}} = 3x^7$

$$\sec^{-1}(3x^7) + C$$

20. $\frac{d}{dx} \tan(e^x)$

$$e^x \sec^2(e^x)$$

21. $\frac{d}{dx} \sec^{-1}(3 \ln x)$

$$\frac{1}{3 \ln x \sqrt{9(\ln x)^2 - 1}} \cdot \frac{3}{x}$$

$$\frac{1}{x \ln x \sqrt{9(\ln x)^2 - 1}}$$

22. $\frac{d}{dx} \sin^{-1}(\sin x)$

$$1$$

23. $\frac{d}{dx} \frac{15x^3 + 3x^2 + 55x}{3x}$
 $\frac{d}{dx} \left(\frac{15x^2}{3x} + \frac{3x^2}{3x} + \frac{55x}{3x} \right)$
 $\frac{d}{dx} \left(5x^2 + x + \frac{55}{3} \right)$

$$10x + 1$$

24. Anti-derivative of
 $f'(x) = -\frac{8x}{\sqrt{1-16x^4}}$

$$\cos^{-1}(4x^2) + C$$

INVERSE FUNCTIONS:

25. If $f(x) = 3x^2$ and $f^{-1}(27) = 3$, find the derivative of $f^{-1}(x)$ at $x = 27$

$$f'(x) = 6x$$

$$\frac{1}{f'[f^{-1}(27)]} = \frac{1}{f'(3)} = \frac{1}{18}$$

26. If $f(x) = \cos 3x$ and $f^{-1}(0) = \frac{\pi}{6}$, find the derivative of $f^{-1}(x)$ at $x = 0$

$$-\frac{1}{3}$$

27. If $f(x) = x^2 + x$ and $f^{-1}(2) = -2$, find the derivative of $f^{-1}(x)$ at $x = 2$

$$f'(x) = 2x + 1$$

$$\frac{1}{f'[f^{-1}(2)]} = \frac{1}{f'(-2)} = -\frac{1}{3}$$

28. If $f(x) = 6x - 2$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 16$

$$\frac{1}{6}$$

29. If $f(x) = \frac{\sqrt{x}}{3}$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 1$

$$x = \frac{9}{3}$$

$$3x = \sqrt{9}$$

$$9x^2 = f^{-1}(x)$$

$$9 = f^{-1}(1)$$

$$f'(x) = \frac{1}{6\sqrt{x}}$$

$$\frac{1}{f'[f^{-1}(1)]} = \frac{1}{f'(9)}$$

$$18$$

30. If $f(x) = 3x^3 - 4$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 20$

$$\frac{1}{36}$$

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions

$$g(1) = 2, \text{ so } g^{-1}(2) = 1.$$

$$(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$$

$$\text{An equation of the tangent line is } y - 1 = \frac{1}{5}(x - 2).$$

Points

$$3 : \begin{cases} 1 : g^{-1}(2) \\ 1 : (g^{-1})'(2) \\ 1 : \text{tangent line equation} \end{cases}$$