Calculus

Write your questions and thoughts here!

6.2 Related Rates

Name:__________________________

Notes

Guidelines to solving related rate problems

1. Draw a picture.
2. Make a list of all known and unknown rates and quantities.
3. Relate the variables in an equation.
4. Differentiate with respect to time.
5. Substitute the known quantities and rates and solve.

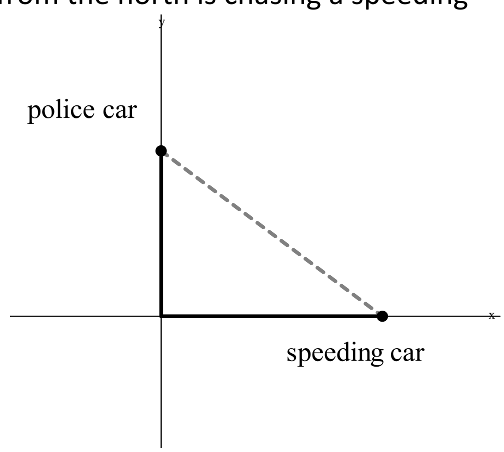
IMPORTANT: Substituting a non-constant quantity before differentiating is not allowed!

Easy example #1

The width of a rectangle is increasing at a rate of 2 cm/sec and its length is increasing at a rate of 3 cm/sec. At what rate is the area of the rectangle increasing when its width is 4 cm and its length is 5 cm?

Easy example #2

A police car, approaching a right-angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. When the police car is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with a radar gun that the distance between them and the car is increasing at 20 mph. If the police car is moving at 60 mph at the instant of measurement, what is the speed of the car?



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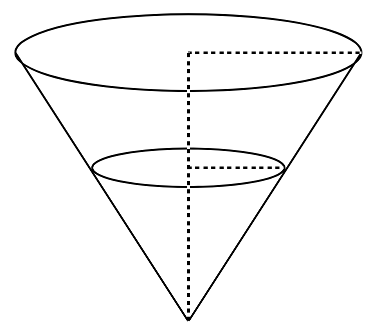
Angle of elevation example

A rocket lifts off at the Kennedy Space Center in Florida. A camera is placed 2000 feet away from the launch pad to film the rocket's ascent. The height of the rocket can be found using $s(t) = 50t^2$, where s is feet and t is seconds. Find the rate of change in the angle of elevation of the camera at 10 seconds after lift-off.

Cone example

Water runs into a conical tank at the rate of 9 ft^3 / min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep? Volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



Now summarize what you learned!

6.2 Related Rates

Calculus Name: ______________________________ **Practice**

1. If $y = 3x^4 + 6x$, find $\frac{dy}{dt}$ when $x = 1$, and $\frac{dx}{dt} = -3$.

Answer: $\frac{dy}{dt} = -54$

2. If $g = 5h - 2h^5$, find $\frac{dg}{dt}$ when $h = 2$, and $\frac{dh}{dt} = 3$.

Answer: $\frac{dg}{dt} = -465$

3. If $x^2 + y^2 = z^2$, find $\frac{dy}{dt}$ when $x = 3$, $y = 4$, $\frac{dx}{dt} = -1$, and $\frac{dz}{dt} = 5$.

Answer: $\frac{dy}{dt} = 7$

4. If $A = \frac{1}{2}bh$, find $\frac{dA}{dt}$ when $b = 7$, $h = 6$, $\frac{db}{dt} = 2$, and $\frac{dh}{dt} = -3$.

Answer: $\frac{dA}{dt} = -\frac{9}{2}$

5. An ice cube is melting at a rate of 5 cubic cm per hour. At what rate is the edge of the cube changing when the edge of the cube is 3 cm.

Answer: $-\frac{5}{27}$ cm/hour

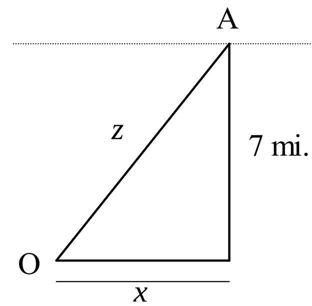
6. A circular pool of water is expanding at the rate of 16π in^2 / sec. At what rate is the radius expanding when the radius is 4 inches?

Answer: 2 inches/sec

7. A spherical balloon is expanding at a rate of 60π in^3 / sec. How fast is the surface area of the balloon expanding when the radius of the balloon is 4 inches? $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.

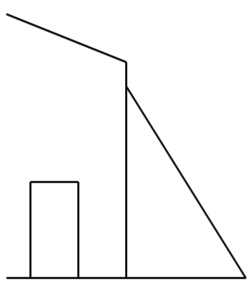
Answer: 30π in^2/sec

8. An airplane (pt. A) is flying 600 mph on a horizontal path that will take it directly over an observer (pt. O). The airplane maintains a constant altitude of 7 miles (see figure). What is the rate of change of the distance between the observer and the airplane when $x = 5$ miles?



Answer: $-\frac{3000}{\sqrt{74}}$ mph

9. Mr. Brust is using a ladder to paint his house. The 17-ft ladder is leaning against the house when Mr. Kelly decides to pull the base of the ladder away from the house at a rate of 3 ft./sec. How fast is the top of the ladder moving down the side of the house when it is 8 ft. above the ground? Indicate units of measure.



Answer: $\frac{45}{8}$ feet/sec

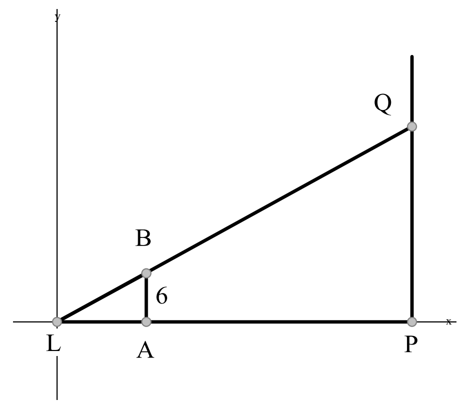
10. A rocket is rising vertically at a rate of 5,400 miles per hour. An observer on the ground is standing 20 miles from the rocket's launch point. How fast (in radians per second) is the angle of elevation between the ground and the observer's line of sight of the rocket increasing when the rocket is at an elevation of 40 miles? (Notice that velocity is given in miles per hour and the answer asks for radians per second.)

Answer: $\frac{3}{200}$ radians / sec

11. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock. The pulley is 7 feet higher than the boat's bow. If the rope is hauled in at a rate of 4 feet/sec, how fast is the boat approaching the dock when 25 feet of rope is out?

Answer: $\frac{25}{6}$ feet/sec

12. A man 6 ft. tall is walking away from a spotlight (L) located on the ground. His shadow is cast on a wall 40 ft. from the spotlight. If the man is walking at a rate of 4 ft. per second away from the spotlight determine the rate of change of the shadow (PQ) when he is half way to the wall.



Answer: $-\frac{12}{5}$ ft/sec

13. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(a) 57.60 (b) 57.88 (c) 59.20 (d) 60.00 (e) 67.40

Answer: (a) 57.6 meters/sec

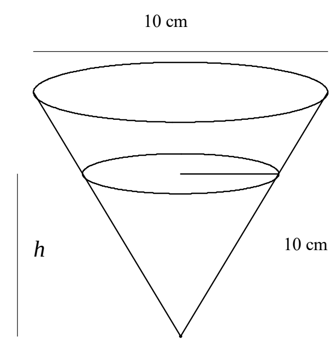
14. The base of a triangle is decreasing at a constant rate of 0.2 cm/sec and the height is increasing at 0.1 cm/sec. If the area is increasing, which answer best describes the constraints on the height h at the instant when the base is 3 centimeters?

(a) $h > 3$ (b) $h < 1$ (c) $h > 1.5$ (d) $h < 1.5$ (e) $h > 2$

Answer: (d) $h < 1.5$

FREE RESPONSE **Your score: _______ out of 9**

2002 AB5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.

Note: the volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.

(a) Find the volume V in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?