9.2 Trig Integrals

Calculus

Name Solutions

Practice

In this practice set you will find definite integrals, indefinite integrals, AND derivatives.

1. $\int (\cos x - 5\sin x)\,dx$

$\boxed{\sin x + 5\cos x + C}$

2. $\int \sec x\,(\sec x + \tan x)\,dx$

$\boxed{\tan x + \sec x + C}$

3. $\int_{\pi/4}^{\pi} 2\cos x\,dx$

$2\sin x \Big|_{\pi/4}^{\pi}$

$2\sin\pi - 2\sin\frac{\pi}{4}$

$2(0) - 2\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{-\sqrt{2}}$

4. $\int_{-3\pi/4}^{-\pi/2} \sin x\,dx$

$\boxed{-\frac{\sqrt{2}}{2}}$

5. $\int_{\pi/9}^{2\pi/9} 3\csc^2 3x\,dx$

$\frac{-3\cot(3x)}{3}\Big|_{\pi/9}^{2\pi/9}$

$-\cot\left(\frac{2\pi}{3}\right) - -\cot\left(\frac{\pi}{3}\right)$

$-\frac{\cos\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)} + \frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}$

$-\frac{\left(-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}} + \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\frac{1}{\frac{\sqrt{3}}{2}} = \boxed{\frac{2}{\sqrt{3}}}$

6. $\int_{\pi/6}^{\pi/4} \csc 2x \cot 2x\,dx$

$\boxed{-\frac{1}{2} + \frac{1}{\sqrt{3}}}$

7. $\int \frac{3}{|x|\sqrt{36x^6 - 1}}\,dx$

$(6x^3)^2$

$\boxed{\sec^{-1}(6x^3) + C}$

8. $\int -\frac{2}{4x^2 + 1}\,dx$

$\boxed{\cot^{-1}(2x) + C}$

9. $\frac{d}{dx}\sin 5x$

$\boxed{5\cos(5x)}$

10. $\frac{d}{dx}\sec^2 2x$

$\boxed{4\sec^2(2x)\tan(2x)}$

11. $\int (\sec^2 x + x)\,dx$

$\boxed{\tan x + \frac{x^2}{2} + C}$

12. $\int \frac{\sin x}{\cos^2 x}\,dx$

$\boxed{\sec x + C}$

13. $\int_0^{\pi} \sec x \tan x \, dx$

$\sec x \Big|_0^{\pi}$

$\sec(\pi) - \sec(0)$

$\frac{1}{\cos\pi} - \frac{1}{\cos 0}$

$\frac{1}{-1} - \frac{1}{1}$

$\boxed{-2}$

14. $\int_{-\pi/4}^{\pi} \sin 2x \, dx$

$\boxed{-\frac{1}{2}}$

15. $\int \frac{20x^4}{\sqrt{1-16x^{10}}} dx$

$(4x^5)^2$

$\boxed{\sin^{-1}(4x^5) + C}$

16. $\int \frac{\cos^3 x + 4}{\cos^2 x} dx$

$\boxed{\sin x + 4\tan x + c}$

17. $\int x - \frac{2}{\cos^2 x} dx$

$\int x - 2\sec^2 x \, dx$

$\boxed{\frac{x^2}{2} - 2\tan x + C}$

18. $\int \frac{72x^7}{1+81x^8} dx$

$\boxed{\tan^{-1}(9x^4) + C}$

19. $\int \frac{1}{\csc x} dx$

$\int \sin dx$

$\boxed{-\cos x + C}$

20. $\frac{d}{dx} \cos 3x$

$\boxed{-3\sin(3x)}$

21. $\int_{\pi/2}^{\pi/2} \csc(\cot(\sec x)) \, dx$

$\boxed{0}$

The upper boundary and the lower boundary are the same!

22. $\frac{d}{dx} \sec x \tan x$

$\boxed{\sec x \tan^2 x + \sec^3 x}$

23. $\int_{\pi/4}^{5\pi/4} \sec^2 x \, dx$

$\tan x \Big|_{\pi/4}^{5\pi/4}$

$\tan\left(\frac{5\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)$

$\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} - \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{0}$

24. $\int \frac{\sin 2x}{\cos x}\,dx$

Hint: $\sin 2x = 2\sin x \cos x$

$\boxed{-2\cos x + C}$

25. $\int_{\pi}^{\frac{\pi}{2}} 3\sin 5x\,dx = -\int_{\frac{\pi}{2}}^{\pi} 3\sin(5x)$

$\frac{3\cos(5x)}{5}\Big|_{\pi/2}^{\pi}$

$\frac{3\cos(5\pi)}{5} - \frac{3\cos(\frac{5\pi}{2})}{5}$

$\frac{3(-1)}{5} - \frac{3(0)}{5} = \boxed{-\frac{3}{5}}$

Test Prep: 1B, 2A, 3D, 4C

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions	Points
(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.	1 : answer with justification
(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint. $f(0) = f(8) + \int_8^0 f'(x)\,dx$ $= f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$ $f(6) = f(8) + \int_8^6 f'(x)\,dx$ $= f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$ $f(8) = 4$ The absolute minimum value of f on the closed interval $[0, 8]$ is -8.	3 : { 1 : considers $x = 0$ and $x = 6$ 1 : answer 1 : justification
(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.	2 : { 1 : answer 1 : explanation
(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$ $g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$	3 : { 2 : $g'(x)$ 1 : answer