Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 1 – Limits [BEAN]			
1.1	Limits Graphically	Define a limit (y-value a function approaches) One-sided limits. Easy if it's continuous. Tricky if there's a discontinuity.	**EK 1.1A1**: Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$
			EK 1.1A3: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
1.2	Limits Analytically	First, attempt direct substitution Other strategies (factoring, radical conjugates) [Don't spend too much time on factoring and simplifying, because we will later use L'Hopitals rule for indeterminate form.] Special Limits: Sine and Cosine special limits. Also taking limits with things like $\frac{\tan 4x}{6x} = \frac{2}{3}$.	**EK 1.1B1**: Numerical and graphical information can be used to estimate limits.
			EK 1.1C1: Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
			EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
1.3	Asymptotes (Limits Involving Infinity)	As x approaches infinity (horizontal asymptotes). To find horizontal asymptotes, you must approach both negative and positive infinity. You will get a different answer when dealing with things like e^x. As x approaches a constant (vertical asymptote) Squeeze Theorem End-behavior model. Show examples of finding the asymptote when a radical is in the numerator but x^2.	**EK 1.1A2**: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
			EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
			EK 1.1D1: Asymptotic and unbounded behavior of functions can be explained and described using limits.

1.4	Continuity (Defined by Limits)	Removable Discontinuity (hole, point discontinuity) Nonremovable Discontinuity (jump, vertical asymptote) Intermediate Value Theorem (for continuous functions) Find the domain of functions (REVIEW) Focus on denominators and radicals!	**EK 1.2A1**: A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.
			EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
			EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
			EK 1.2B1: Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
			EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 2 – The Derivative [BEAN]			
2.1	Local Linearity	Dive into "Average Rate of Change" and make the intervals smaller and smaller. If we zoom in on any "smooth" graph, it will look like a line, no matter how sharp it turns. Linear Approximation	**EK 2.1A1**: The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
			EK 2.1B1: The derivative at a point can be estimated from information given in tables or graphs.
			EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
2.2	Def. of Derivative	Use "Delta X" and "h"? Be able to see the Def. of Der. (limit notation) and recognize what the original function was. Come back to that concept after they learn power rule and chain rule. Harp on the phrases: • Slope of the tangent line. • Instantaneous rate of change. • Slope at a point.	**EK 2.1A2**: The instantaneous rate of change of a function at a point can be expressed by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
			EK 2.1A3: The derivative of f is the function whose value at x is $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ provided this limit exists.
			EK 2.1A4: For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.

		Include application problems where students describe the meaning of something like $f'(3) = 10$ and $f(3) = 5$ in several scenarios. Have student graph a function (parabola, cubic, square root). Find the derivative. Graph the tangent line at a given point.	**EK 2.1A5**: The derivative can be represented graphically, numerically, analytically, and verbally.
			EK 2.1C1: Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
			EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
			EK 2.3B1: The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
			EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.
2.3	Graphs of f and f'	Relationships between f and f' Matching graphs of f and f'. [One of the advantages of doing it this early is it drills into the kids' head that the derivative represents the slope of the function.]	**EK 2.1A5**: The derivative can be represented graphically, numerically, analytically, and verbally.
			EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
			EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.
2.4	Differentiability [Calculator Required]	Differentiability implies continuity. Include piecewise functions and show continuous and differentiable. This really needs to be covered in Unit 3 after they have learned Power Rule. Mean Value Theorem for derivatives. [Currently it is in lesson 5.2] NDER operation on the calculator.	**EK 2.1A4**: For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.
			EK 2.2B1: A continuous function may fail to be differentiable at a point in its domain.
			EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.
			EK 2.3A1: The unit for $f'(x)$ is the unit for f divided by the unit for x.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 3 – Basic Differentiation [BRUST]			
3.1	Power Rule	Sum and differences should be included (multiple terms). Higher Order derivatives. Find equations of the tangent line and normal line. (both point-slope and slope-intercept)	**EK 2.1C2**: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
			EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
			EK 2.1D1: Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
			EK 2.1D2: Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.
			EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.
3.2	Product and Quotient Rules	Use the same order for product rule that we use for the quotient rule. (explain the difference that kids might see in books. Addition is commutative). Find equations of the tangent line and normal line.	**EK 2.1C3**: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
3.3	Linear Velocity	Position, Velocity, acceleration, speed, and other rates.	**EK 2.3C1**: The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
			EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.
3.4	Chain Rule	Derivatives of Composite functions (no trig yet)	**EK 2.1C4**: The chain rule provides a way to differentiate composite functions.
3.5	Trig Derivatives	All 6 trig derivatives. Include chain rule for most problems. Find equations of the tangent line and normal line.	**EK 2.1C4**: The chain rule provides a way to differentiate composite functions.
			EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 4 – Derivatives (Exp, Logs, Inverse Trig) [BEAN]			
4.1	Derivatives of Exponential and Logarithmic Functions		**EK 2.1C2**: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
4.2	Inverse Trig Derivatives	Trick to remember the Inverse trig derivatives. Derivative of inverse functions (Notation: arcsine and $\sin^{-1} x$)	**EK 2.1C2**: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric. **EK 2.1C6**: The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
4.3	Limits Revisited: L'Hopital's Rule	This will be an easy, brief lesson. (4-page packet is probably long enough) Trig Limits revisited?	**EK 1.1C3**: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 5 – Curve Sketching [BRUST]			
5.1	Extrema on an Interval Points	Local and absolute extrema. Extreme Value Theorem Finding Critical Points (candidates for extrema)	**EK 1.2B1**: Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
5.2	1st Derivative Test	Mean Value Theorem (average value of the derivative) [Should we put the MVT in an earlier lesson? This is a lot for one lesson.] Increasing and Decreasing Functions First Derivative Test	**EK 1.2B1**: Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem. **EK 2.2A1**: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection. **EK 2.4A1**: If a function f is continuous over the interval and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

5.3	2nd Derivative Test	2nd Derivative Test	**EK 2.2A1**: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
	Matching activity [This will be the same day as the Review Day. Students must come in with one other student and put together matching cards.]	Match the following items: • Function's equation • Function's graph • 1st derivative graph • 2nd derivative graph	**EK 2.2A1**: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
			EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 6 – Implicit Differentiation [BEAN]			
6.1	Implicit Differentiation		**EK 2.1C5**:The chain rule is the basis for implicit differentiation.
6.2	Related Rates	This is very challenging. Do we need two days?	**EK 2.3C2**: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
			EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.
6.3	Optimization	Be sure to have problems that ask for the maximum value of the function so that kids must pay attention to the wording. For example, give the rate function and ask for the fastest rate. Linear velocity revisited.	**EK 2.3C3**: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
PRACTICE TEST!!! (AP style exam that covers derivatives) This is a timed test.			

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)

Unit 7 – Approximation Methods (area under the curve) [BEAN]			
7.1	Rectangular Approximation Method	Left, Right, and Midpoint Approximations (Riemann Sums) Minimum Approximations vs. Maximum Approximations Use Tables for Free Response problems	**EK 3.2A1**: A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
			EK 3.2B2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
7.2	Trapezoidal Approximation	Complicated formula from the book. There's an easier way to do it! Use Tables for Free Response problems.	**EK 3.2B2**: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.

Unit 8 – Integration [BRUST]			
8.1	Definite Integral	Show the meaning of a definite integral graphically. (area under the curve). Instead of an "approximation method" we are finding the exact area. Properties of Integrals (constants, reversing bounds) Use a calculator to evaluate a definite integral.	**EK 3.2C2**: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
			EK 3.2B1: Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
			EK 3.2C3: The definition of the definite integral may be extended to functions with removable or jump discontinuities.
			EK 3.2A3: The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
			EK 3.4A3: The limit of an approximating Riemann sum can be interpreted as a definite integral.
			EK 3.2C1: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
8.2	The Fundamental Theorem of Calculus (part 1)	$\int_a^b f(x)\,dx = F(a) - F(b)$ Basic Antiderivatives	**EK 3.3B2**: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

			EK 3.2A2: The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)\,dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)\,dx = \lim_{\max\Delta x_i \to 0} \sum_{i=1}^{n} f({x_i}^*)\Delta x_i$ where ${x_i}^*$ is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and $\max\Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f({x_i}^*)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and ${x_i}^*$ is a value in the ith subinterval.
8.3	Anti-Derivatives	More FTC (part 1) with more challenging anti-differentiation. Indefinite Integration (+ C at the end) Finding specific solutions to a differential equation.	**EK 3.5A1**: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay.
			EK 3.1A1: An antiderivative of a function f is a function g whose derivative is f.
			EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives.
			EK 3.3B3: The notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function f.
			EK 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
			EK 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation.
			EK 3.3B4: Many functions do not have closed form antiderivatives.
			EK 2.3E1: Solutions to differential equations are functions or families of functions.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 9 – FTC part 2 [BEAN]			
9.1	Fundamental Theorem of Calculus (part 2)	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \bullet x'$ Include functions defined by an integral with the upper boundary as "x".	**EK 3.3A1**: The definite integral can be used to define new functions; for example, $f(x) = \int_0^x e^{-t^2}\,dt$.
			EK 3.3A2: If f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, where x is between a and b.
			EK 3.3B1: The function defined by $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f.
			EK 3.5A4: The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.
9.2	Trig Integrals	Trig integrals and inverse trig integrals. (Notation: arcsine and $\sin^{-1} x$)	NONE!! Which is strange, because this is tested on the AP exam.
9.3	Integrals using Geometry	[Check with Brust so that this does not overlap what was done in 8.1. There will be similarities, but the focus should be on FTC part 2.]	**EK 3.2C1**: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
			EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f

			provide information about the function g defined as $g(x) = \int_a^x f(t)\,dt$. .
9.4	Net Change	Displacement vs. total distance traveled. Include problems dealing with accumulation. Net Change	**EK 3.4A1**: A function defined as an integral represents an accumulation of a rate of change.
			EK 3.4A2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
			EK 3.4C1: For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
			EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many applied contexts.

Unit/ Lesson	Topic	Subtopics	Essential Knowledge (EK)
Unit 10 – More Integrals [BRUST]			
10.1	Slope Fields	Formally explain "Differential Equations," even though we've been using them for several lessons already. [Should we have something on slope fields earlier?]	**EK 2.3F1**: Slope fields provide visual clues to the behavior of solutions to first order differential equations.
10.2	u-substitution (indefinite integrals	Reverse of Chain Rule. If chain rule were needed for the derivative, then u-substitution is needed for the integral.	**EK 3.3B5**: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
10.3	u-substitution (definite integrals)	10.2 and 10.3 could be combined, but this topic is so important, the extra practice is good for one more day. Introduce the following special case integrals: $\int \frac{1}{x}\,dx = \ln x + C$ $\int a^x\,dx = \frac{a^x}{\ln a} + C$	**EK 3.3B5**: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.

		$\int e^x\,dx = e^x + C$	
10.4	Separation of Variables	Separation of Variables Exponential Growth and Decay	**EK 3.5B1**: The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.
			EK 3.5A2: Some differential equations can be solved by separation of variables.
			EK 3.5A3: Solutions to differential equations may be subject to domain restrictions.
10.5	Average Value (of a function)	Compare and distinguish between Average Value of a function, Mean Value Theorem (average derivative), and Average Rate of Change.	**EK 3.4B1**: The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.

Unit 11 – Area and Volume **[BEAN or BRUST?]**			
11.1	Area Between Two Curves		**EK 3.4D1**: Areas of certain regions in the plane can be calculated with definite integrals.
11.2	Volume - Disc Method		**EK 3.4D2**: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
11.3	Volume - Washer Method		**EK 3.4D2**: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
11.4	Perpendicular Cross Sections		**EK 3.4D2**: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
Practice Test!! Timed Test			