Unit 5 Curve Sketching

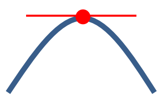
REVIEW

DEFINTIONS

Extrema: The maximum and minimum points. Extrema can be absolute or relative.

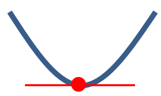
Critical Points: Where the first derivative is zero or DNE. Possible maximum, minimum, or point of inflection!

$f'(x) = 0$



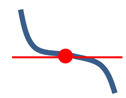
Horizontal Tangent
Maximum

$f'(x) = 0$



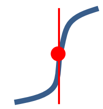
Horizontal Tangent
Minimum

$f'(x) = 0$



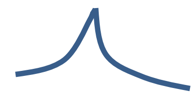
Horizontal Tangent
Point of Inflection

$f'(x) = DNE$



Vertical Tangent
Point of Inflection

$f'(x) = DNE$



Cusp (No Tangent)
Maximum

Concavity: Where the function is “cupping” up or down

CONCAVE UP

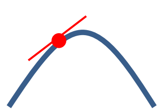
CONCAVE DOWN

Points of Inflection: Where the second derivative is zero or DNE and changes in concavity!

First Derivative

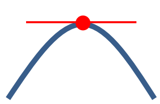
The first derivative is the instantaneous rate of change, the slope of tangent line and can determine if the function is increasing or decreasing at a given point.

$f'(x) > 0$



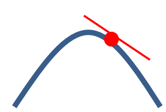
Function is increasing

$f'(x) = 0$



Function is not increasing or decreasing

$f'(x) < 0$



Function is decreasing

Second Derivative

The second derivative determines concavity.

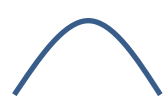
$f''(x) > 0$

Concave Up

$f''(x) = 0$

Neither concave up or concave down

$f''(x) < 0$



Concave Down

FINDING EXTREMA

First Derivative Test

STEPS	EXAMPLE
	$f(x) = x^2 + 2x + 1$
1. Find the critical points.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$

2. Determine whether the function is increasing or decreasing on each side of every critical point. A chart or number line helps!

Interval	$(-\infty, -1)$	-1	$(-1, \infty)$
Test Value	-2	-1	2
$f'(x)$	$f'(-2) = -2$ Negative	$f'(-1) = 0$	$f'(2) = 6$ Positive

Function decreases to the left and increases to the right of $x = -1$ so it must be relative minimum point

Second Derivative Test

STEPS	EXAMPLE
	$f(x) = x^2 + 2x + 1$
1. Find the critical points.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Determine whether the function is concave up or concave down at every critical point using the second derivative.	$f''(-1) = 2$ Second derivative is positive at $x = -1$ Concave up $x = -1$ is a relative minimum point

Finding Absolute Extrema on an interval

STEPS	EXAMPLE
	$f(x) = x^2 + 2x + 1$ on the interval $[-3,0]$
1. Find the critical points. The critical points are candidates as well as the endpoints of the interval.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Check all candidates using the $f(x)$.	$f(-3) = 4$ absolute maximum $f(-1) = 0$ absolute minimum $f(0) = 1$

LINEAR MOTION (PARTICLE MOTION)

The chart matches up function vocab with linear motion vocab.

FUNCTION	LINEAR MOTION
Value of a function at *x*	Position at time *t*
First Derivative	Velocity
Second Derivative	Acceleration
$f'(x) > 0$ Increasing Function	Moving right or up
$f'(x) < 0$ Decreasing Function	Moving left or down
$f'(x) = 0$	Not moving
Absolute Max	Farthest right or up
Absolute Min	Farthest left or down
$f'(x)$ changes signs	Object changes direction
$f'(x)$ and $f''(x)$ have same sign	Speeding Up
$f'(x)$ and $f''(x)$ have different signs	Slowing Down

Example:

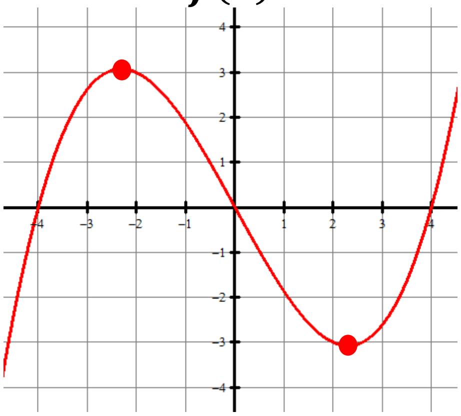
A particle moves along the *x*-axis with the position function $x(t) = t^4 - 4t^3 + 2$ where $t > 0$.

Interval	$(0, 2)$	**2**	$(2, 3)$	**3**	$(3, \infty)$
$f'(x)$ **velocity**	$f'(x) > 0$ increasing right	$f'(x) > 0$ increasing right	$f'(x) > 0$ increasing right	$f'(x) = 0$ Not moving	$f'(x) < 0$ decreasing left
$f''(x)$ **acceleration**	$f''(x) > 0$ Concave up	$f''(x) = 0$	$f''(x) < 0$ Concave down	$f''(x) < 0$ Concave down	$f''(x) < 0$ Concave down
Conclude	Speeding Up	Moving Right	Slowing Down	Not Moving	Speeding Up

FUNCTION	LINEAR MOTION
$t = 3$ is maximum	$t = 3$ has no velocity Changing direction
Increasing $(0,3)$	Moving right $(0,3)$
Decreasing $(3, \infty)$	Moving left $(3, \infty)$

Graphical Analysis

$f(x)$

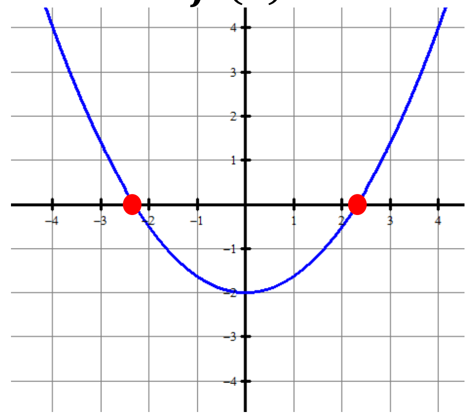


$f(x)$ max at $x = -2.2$ and min $x = 2.2$
So $f'(x) = 0$ at these points

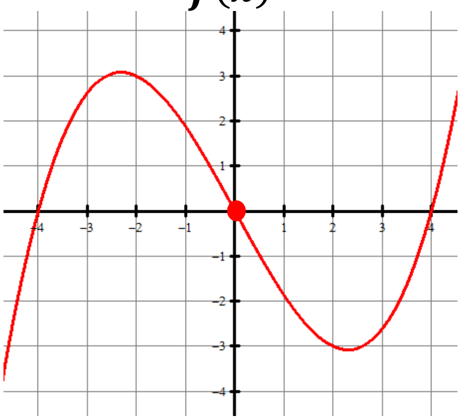
$f(x)$ is increasing $(-\infty, -2.2)(2.2, \infty)$
So $f'(x) > 0$ on these intervals

$f(x)$ is increasing $(-\infty, -2.2)(2.2, \infty)$
So $f'(x) > 0$ on these intervals

$f'(x)$



$f(x)$

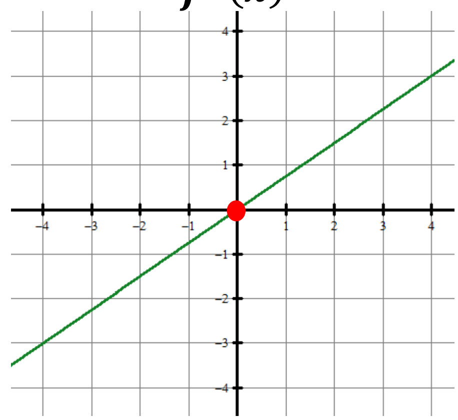


$f(x)$ is concave down $(-\infty, 0)$
So $f''(x) < 0$ on this interval

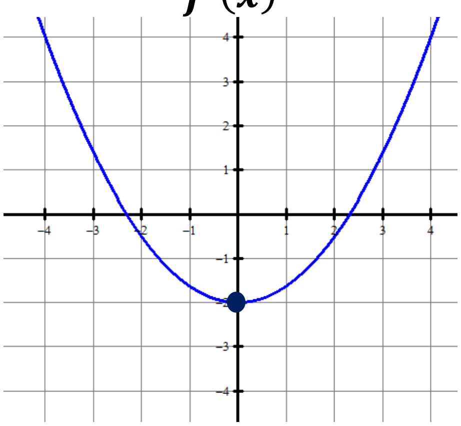
$f(x)$ is concave up $(0, \infty)$
So $f''(x) > 0$ on this interval

$x = 0$ is a point of inflection on $f(x)$
So $f''(x) = 0$ at $x = 0$

$f''(x)$



$f'(x)$



$f'(x)$ min $x = 0$
So $f''(x) = 0$ at $x = 0$

$f'(x)$ is decreasing $(-\infty, 0)$
So $f''(x) < 0$ on this interval

$f'(x)$ is increasing $(0, \infty)$
So $f''(x) > 0$ on this interval

$f''(x)$