5.9 Connecting f, f', and f''

Calculus

Practice

1. A particle's position along the x-axis is measured by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 1$ where $t > 0$. Find the intervals where the particle is speeding up. Find intervals where the particle is slowing down.

$V(t) = t^2 - 6t + 8$

$(t-4)(t-2) = 0$

$t = 4 \quad t = 2$

$a(t) = 2t - 6 = 0$

$2t = 6$

$t = 3$

t	$(0,2)$	2	$(2,4)$	4	$(4,\infty)$
$V(t)$	$+$	0	$-$	0	$+$

t	$(0,3)$	3	$(3,\infty)$
$a(t)$	$-$	0	$+$

$\boxed{\text{Speeding up: } (2,3) \text{ and } (4,\infty) \quad \text{Slowing down: } (0,2) \text{ and } (3,4)}$

2. A particle's position along the y-axis is measured by $y(t) = t - 3(t-4)^{\frac{1}{3}}$ where $t > 0$. Find the intervals where the particle is speeding up. Find intervals where the particle is slowing down.

$V(t) = 1 - (t-4)^{-2/3} = 0$

C.P. at $t = 4$

$\frac{1}{(t-4)^{2/3}} = 1$

$1 = (t-4)^{2/3}$

$\pm 1 = t - 4$

$t = 3$ and $t = 5$

$a(t) = \frac{2}{3}(t-4)^{-5/3}$

Possible point of inflection at $t = 4$

t	$(0,3)$	3	$(3,4)$	4	$(4,5)$	5	$(5,\infty)$
$V(t)$	$+$	0	$-$	und	$-$	0	$+$
$a(t)$	$-$	$-$	$-$	und	$+$	$+$	$+$

$\boxed{\text{Speeding up: } (3,4) \text{ and } (5,\infty) \quad \text{Slowing down: } (0,3) \text{ and } (4,5)}$

For each table, selected values of x and $f(x)$ are given. Assume that $f'(x)$ and $f''(x)$ do not change signs. Answer the questions for each table.

3.

x	$f(x)$
4	−5
5	−8
6	−12
7	−17

−3, −4, −5

a. Is $f(x)$ increasing or decreasing?

Decreasing

b. Is $f(x)$ concave up or concave down?

Concave Down

4.

x	$f(x)$
−3	−2
−2	3
−1	7
0	10

5, 4, 3

a. Is $f(x)$ increasing or decreasing?

Increasing

b. Is $f(x)$ concave up or concave down?

Concave Down

5.

x	$f(x)$
2	3
3	0
4	−2
5	−3

−3, −2, −1

a. Is $f(x)$ increasing or decreasing?

Decreasing

b. Is $f(x)$ concave up or concave down?

Concave Up

6. Given the function $g(x) = -x^4 + 2x^2 - 1$, find the interval(s) when g is concave up and increasing at the same time.

$g'(x) = -4x^3 + 4x$

$-4x(x^2-1) = 0$

$x=0, \quad x=-1, \quad x=1$

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
g'	+	0	−	0	+	0	−
	inc				inc		

$g''(x) = -12x^2 + 4 = 0$

$-12x^2 = -4$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

x	$(-\infty,-\sqrt{\frac{1}{3}})$	$-\sqrt{\frac{1}{3}}$	$(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}})$	$\sqrt{\frac{1}{3}}$	$(\sqrt{\frac{1}{3}},\infty)$
g''	−	0	+	0	−
			up		

answer: $\left(0, \frac{\sqrt{3}}{3}\right)$

$\boxed{\left(0, \sqrt{\frac{1}{3}}\right)}$

7. Given the function $h(x) = x^3 - 2x^2 + x$, find the interval(s) when h is concave up and decreasing at the same time.

$h'(x) = 3x^2 - 4x + 1$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3} \qquad x = 1$

x	$(-\infty,\frac{1}{3})$	$\frac{1}{3}$	$(\frac{1}{3},1)$	1	$(1,\infty)$
h'	+	0	−	0	+
			decreasing		

$h''(x) = 6x - 4 = 0$

$x = \frac{2}{3}$

x	$(-\infty,\frac{2}{3})$	$\frac{2}{3}$	$(\frac{2}{3},\infty)$
h''	−	0	+
			up

$\boxed{\left(\frac{2}{3}, 1\right)}$

8. **Calculator active problem.** Let h be the function given by $h(t) = 70 - 15\cos\left(\frac{\pi t}{3}\right) + 5\sin\left(\frac{\pi t}{4}\right)$ for $0 \leq t \leq 5$. At what value of t is h increasing most rapidly?

→ when is h' a maximum?

(A) 0.266 (B) 1.343 (C) 2.851 (D) 4.439 (E) 5.000

9.

x	-5	-4	-2	0	3
$f'(x)$	-8	-10	-7	-4	-6

Calculator active problem. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-5 < x < 3$?

(A) The graph of f has at least two points of inflection.

(B) The graph of f is concave down.

(C) f is decreasing.

(D) f has at least two relative extrema.

(E) f has no critical points.

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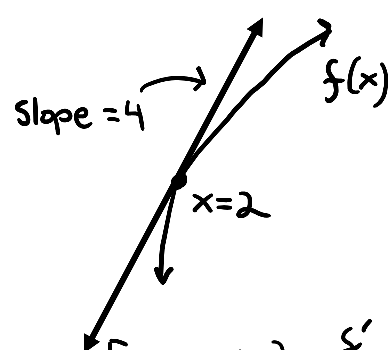
10. In the xy-plane, the graph of the twice-differentiable function $y = f(x)$ is concave down on the open interval $(1, 3)$ and is tangent to the line $y = 4x + 3$ at $x = 2$. Which of the following statements must be true about the derivative of f?



For $x > 2$, f' is getting smaller.

(A) $f'(x)$ is constant on the interval $(2, 2.1)$.

(B) $f'(x) > 0$ on the interval $(2, 2.1)$.

(C) $f'(x) < 0$ on the interval $(2, 2.1)$.

(D) $f'(x) \geq 4$ on the interval $(2, 2.1)$.

(E) $f'(x) \leq 4$ on the interval $(2, 2.1)$.