

# 1.12 Confirming Continuity Over an Interval

# Solutions

# Practice

Calculus

Find the domain of each function.

1.  $g(x) = \sqrt{12 - 2x}$

$$12 - 2x \geq 0$$

$$-2x \geq -12$$

$$x \leq 6$$

or

$$(-\infty, 6]$$

2.  $f(x) = \frac{x-7}{x^2-9x+14} = \frac{x-7}{(x-7)(x-2)}$

$$\mathbb{R}, x \neq 7, x \neq 2$$

or

$$(-\infty, 2) \cup (2, 7) \cup (7, \infty)$$

3.  $f(x) = \ln(2x + 5)$

$$2x + 5 > 0$$

$$x > -\frac{5}{2}$$

or

$$(-\frac{5}{2}, \infty)$$

4.  $h(x) = \frac{5-x}{5-\sqrt{x}}$

$$5 - \sqrt{x} \neq 0$$

$$-\sqrt{x} \neq -5$$

$$x \geq 0, x \neq 25$$

or

$$[0, 25) \cup (25, \infty)$$

5.  $h(t) = \frac{\sqrt{t-1}}{t^2-2t-48} = \frac{\sqrt{t-1}}{(t+6)(t-8)}$

$$t-1 \geq 0$$

$$t \neq -6$$

$$t \geq 1$$

$$t \neq 8$$

$$t \geq 1 \text{ and } t \neq 8$$

or

$$[1, 8) \cup (8, \infty)$$

6.  $h(x) = \frac{5-x}{\sqrt{5-x}}$

$$\sqrt{5-x} \neq 0$$

$$5-x \geq 0$$

$$x \neq 5$$

$$x \leq 5$$

$$x < 5$$

or

$$(-\infty, 5)$$

7.  $h(x) = \ln\left(\frac{x}{x-10}\right)$

$$\frac{x}{x-10} > 0 \rightarrow x \neq 10$$

if  $x < 0$  you get  $\frac{\text{neg.}}{\text{neg.}}$

if  $x > 10$  you get  $\frac{\text{pos.}}{\text{pos.}}$

$$x < 0 \text{ and } x > 10$$

or

$$(-\infty, 0) \cup (10, \infty)$$

8.  $w(t) = \frac{t^2-5}{2}$

$\mathbb{R}$   
"all real numbers"

or

$$(-\infty, \infty)$$

9.  $h(x) = \frac{\sqrt{x-5}}{x-3}$

$$x-5 \geq 0$$

$$x-3 \neq 0$$

$$x \geq 5, x \neq 3$$

~~$x \neq 3$~~

not needed

or

$$[5, \infty)$$

10.  $f(x) = \frac{x+8}{x^2+8x} = \frac{x+8}{x(x+8)}$

$\mathbb{R}$  except  
 $x \neq 0, x \neq -8$

or

$$(-\infty, -8) \cup (-8, 0) \cup (0, \infty)$$

11.  $g(x) = \ln(\sqrt{x-7})$

$$\sqrt{x-7} > 0$$

$$x-7 > 0$$

$$x > 7$$

or

$$(7, \infty)$$

12.  $g(t) = \sqrt{3-5t}$

$$3-5t \geq 0$$

$$-5t \geq -3$$

$$t \leq \frac{3}{5}$$

or

$$(-\infty, \frac{3}{5}]$$

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## Test Prep

13. Let  $f$  be the function given below. On which of the following intervals is  $f$  continuous?

$$f(x) = \begin{cases} 3^x, & x \leq -1 \\ \frac{2x+3}{x+4}, & -1 < x \leq 0 \\ x^2 + 2x, & 0 < x < 4 \\ \tan(x), & x \geq 4 \end{cases}$$

$3^{-1} = \frac{1}{3} \checkmark$   
 $\frac{2(-1)+3}{(-1)+4} = \frac{1}{3} \checkmark$   
 $\frac{2(0)+3}{(0)+4} = \frac{3}{4}$   
 $0^2 + 2(0) = 0$

*Cont. at  $x=1$*   
*Jump at  $x=4$*   
 $4^2 + 2(4) \neq \tan(4)$

- (A)  $(-5, 0)$  (B)  $(-0.5, 3)$  (C)  $(3, 5)$  (D)  $(5, \infty)$

14. Which of the following functions is not continuous on the interval  $-\infty < x < \infty$ ?

(A)  $f(x) = \cos(x)$

(B)  $g(x) = \frac{1}{1+2^{-x}}$

(C)  $h(x) = \frac{1}{x^6 + x^4 + x - 2}$

(D)  $v(x) = x^6 + x^4 + x - 2$

(C)

15. Which of the following functions are continuous on the interval  $1 < x < 6$ ?

I.  $f(x) = \frac{x-4}{x^2-16}$

II.  $g(x) = \frac{x-4}{x^2+16}$

III.  $h(x) = \ln\left(\frac{1}{x}\right)$

D

(A) II only

(B) I and II only

(C) I and III only

(D) II and III only