1.13 Removing Discontinuities

Calculus

Solutions **Practice**

1. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-9}{x-3}$ when $x \neq 3$, then $f(3) =$

$\frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$ $\lim_{x\to 3} f(x) = \boxed{6}$

2. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+8x-20}{x+10}$ when $x \neq -10$, then $f(-10) =$

$\frac{\cancel{(x+10)}(x-2)}{\cancel{x+10}}$, $\lim_{x\to -10} f(x) = \boxed{-12}$

3. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-5x+4}{x-1}$ when $x \neq 1$, then $f(1) =$

$\frac{\cancel{(x-1)}(x-4)}{\cancel{x-1}}$ $\lim_{x\to 1} f(x) = \boxed{-3}$

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+14x+48}{x+8}$ when $x \neq -8$, then $f(-8) =$

$\frac{(x+8)(x+6)}{x+8}$ $\lim_{x\to -8} f(x) = \boxed{-2}$

5. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2-2x-15}{x-5}, & x \neq 5 \\ a, & x = 5 \end{cases}.$$
For what value of a is f continuous at $x = 5$?

$\frac{\cancel{(x-5)}(x+3)}{\cancel{x-5}}$ $5+3 = \boxed{8}$

6. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2-16x+63}{x-7}, & x \neq 7 \\ b, & x = 7 \end{cases}.$$
For what value of b is f continuous at $x = 7$?

$\frac{\cancel{(x-7)}(x-9)}{\cancel{x-7}}$ $7-9 = \boxed{-2}$

7. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2-8x}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}.$$
For what value of c is f continuous at $x = 0$?

$\frac{\cancel{x}(x-8)}{\cancel{x}}$ $0-8 = \boxed{-8}$

8. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2-8x+15}{x-3}, & x \neq 3 \\ a, & x = 3 \end{cases}.$$
For what value of a is f continuous at $x = 3$?

$\frac{\cancel{(x-3)}(x-5)}{\cancel{x-3}}$ $3-5 = \boxed{-2}$

9. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2+5x+4}{b(x+1)}, & x \neq -1 \\ b, & x = -1 \end{cases}.$$
For what value of b is f continuous at $x = 1$?

$\frac{\cancel{(x+1)}(x+4)}{b\cancel{(x+1)}}$

$\frac{-1+4}{b} = b$

$\frac{3}{b} = b$

$3 = b^2$

$\boxed{b = \pm\sqrt{3}}$

10. Let f be the function defined by
$$f(x) = \begin{cases} \frac{x^2-49}{c(x+7)}, & x \neq -7 \\ c, & x = -7 \end{cases}.$$
For what value of c is f continuous at $x = -7$?

$\frac{(x-7)\cancel{(x+7)}}{c\cancel{(x+7)}}$

$\frac{-7-7}{c} = c$

$\frac{-14}{c} = c$

$-14 = c^2$

$c = \pm\sqrt{-14}$

$\boxed{\text{No such value of } c.}$

11. Let f be the function defined by

$$f(x) = \begin{cases} \frac{\sin(6x)}{5x}, & x \neq 0 \\ a, & x = 0 \end{cases}.$$

For what value of a is f continuous at $x = 0$?

$\lim_{x \to 0} f(x) = 6/5$

$\boxed{a = 6/5}$

12. Let f be the function defined by

$$f(x) = \begin{cases} \frac{5\sin(3x)}{4x}, & x \neq 0 \\ b, & x = 0 \end{cases}.$$

For what value of b is f continuous at $x = 0$?

$\lim_{x \to 0} f(x) = \frac{5 \cdot 3}{4} = 15/4$

$\boxed{b = 15/4}$

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Test Prep

13. Let $y = \frac{x^2+4x-21}{x^2-9}$. This function has a hole. What is the y-value of the hole?

A

$\frac{(x+7)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{x+7}{x+3}$

$\lim_{x \to 3} y = \frac{3+7}{3+3} = 10/6$

(A) $\frac{5}{3}$ (B) 3 (C) $-\frac{10}{3}$ (D) 0 (E) -3

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14. For what value of k will the function $f(x) = \frac{x^2-(k+2)x+6}{x-k}$ have a point discontinuity at $x = k$?

Factor numerator $\rightarrow$ $\frac{(x-k)(x-2)}{x-k}$

E

$(-k)(-2)$ must equal 6

(A) $k = -1$ (B) $k = 0$ (C) $k = 1$ (D) $k = 2$ (E) $k = 3$