1.16 Intermediate Value Theorem (IVT)
Calculus

Solutions

Practice

Below is a table of values for a continuous function f.

x	-5	1	3	8	14
$f(x)$	7	40	21	75	-100

1. On the interval $-5 \le x \le 1$, must there be a value of x for which $f(x) = 30$? Explain.

✓ i. f is continuous on $[-5, 1]$
✓ ii. $f(-5) = 7, f(1) = 40$
$f(-5) \neq f(1)$
✓ iii. $k = 30$ is between $f(-5)$ and $f(1)$.

∴ IVT applies and there exists a value c between $(-5, 1)$ such that $f(c) = 30$.

2. On the interval $-5 \le x \le 14$ what is the minimum number of zeros?

1

3. For $1 \le x \le 14$, what is the fewest possible times $f(x) = 20$?

1

4. For $1 \le x \le 8$, what is the fewest possible times $f(x) = 7$?

0

Below is a table of values for a continuous function h.

x	-7	-2	1	4	11
$h(x)$	2	-5	6	-1	10

5. On the interval $4 \le x \le 11$, must there be a value of x for which $h(x) = -2$? Explain.

✓ i. h is continuous on $[4, 11]$
✓ ii. $h(4) = -1, h(11) = 10$
$h(4) \neq h(11)$
✗ iii. $k = -2$ is **NOT** between $h(4)$ and $h(11)$.

The IVT does not apply. There is no guarantee that $h(x) = -2$.

6. On the interval $-7 \le x \le 11$ what is the minimum number of zeros?

4

7. For $-2 \le x \le 4$, what is the fewest possible times $h(x) = 2$?

2

8. For $-7 \le x \le 1$, what is the fewest possible times $h(x) = 3$?

1

Below is a table of values for a continuous function g.

x	0	2	15	32	50
$g(x)$	-1	10	17	-10	8

9. On the interval $15 \le x \le 32$, must there be a value of x for which $g(x) = 11$? Explain

✓ i. g is continuous on $[15, 32]$
✓ ii. $g(15) = 17, g(32) = -10$
$g(15) \neq g(32)$
✓ iii. $k = 11$ is between $g(15)$ and $g(32)$.

∴ IVT applies and there exists a value c between $(15, 32)$ such that $g(c) = 11$.

10. On the interval $15 \le x \le 50$ what is the minimum number of zeros?

2

11. On the interval $0 \le x \le 50$ what is the minimum number of zeros?

3

12. For $15 \le x \le 50$, what is the fewest possible times $g(x) = 1$?

2

Determine if the Intermediate Value Theorem holds for the given value of k.

13. $f(x) = 3 - x^2, [a,b] = [-2,1], k = 0$

✓ i. f is continuous on $[-2,1]$.

✓ ii. $f(-2) = -1, f(1) = 2$
$f(-2) \neq f(11)$

✓ iii. $k = 0$ is between $f(-2)$ and $f(1)$.

∴ IVT applies and there exists a value c between $(-2,1)$ such that $f(c) = 0$.

14. $g(x) = \frac{1}{x}, [a,b] = [2,5], k = -1$

✓ i. g is continuous on $[2,5]$.

✓ ii. $g(2) = \frac{1}{2}, g(5) = \frac{1}{5}$
$g(2) \neq g(5)$

✗ iii. $k = -1$ is **NOT** between $g(2)$ and $g(5)$.

The IVT does not apply.

15. **Calculator active.**
$h(x) = \ln(2x+1), [a,b] = [2,20], k = 3$

✓ i. h is continuous on $[2,20]$.

✓ ii. $h(2) \approx 1.609, h(20) \approx 3.7135$
$h(2) \neq h(20)$

✓ iii. $k = 3$ is between $h(2)$ and $h(20)$.

∴ IVT applies and there exists a value c between $(2,20)$ such that $h(c) = 3$.

16. $f(t) = 3t^2 - 10t + 2, [a,b] = [-1,3], k = 1$

✓ i. f is continuous on $[-1,3]$.

✓ ii. $f(-1) = 15, f(3) = -1$
$f(-1) \neq f(3)$

✓ iii. $k = 1$ is between $f(-1)$ and $f(3)$.

∴ IVT applies and there exists a value c between $(-1,3)$ such that $f(c) = 1$.

1.16 Intermediate Value Theorem (IVT)

Test Prep

17. Let f be a continuous function such that $f(1) = 7$ and $f(7) = 1$. Let g be the function given by $g(x) = f(x) - x$. Explain why there must be a value c for $1 < c < 7$ such that $g(c) = 0$.

✓ i. $g(x)$ is continuous

✓ ii. $g(1) = f(1) - 1 = 7 - 1 = 6$
$g(7) = f(7) - 7 = 1 - 7 = -6$
$g(1) \neq g(7)$

✓ iii. $k = 0$ is between $g(1)$ and $g(7)$

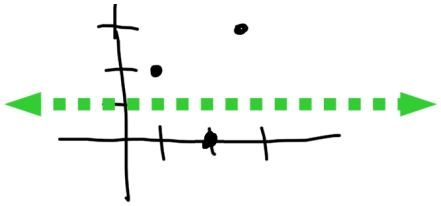
∴ IVT applies and there exists a value c between $(1,7)$ such that $g(c) = 0$.

18. The function f is continuous on the closed interval $[1, 3]$ and has values that are given in the table below.

A

x	1	2	3
$f(x)$	2	k	3

0



The equation $g(x) = 1$ must have at least two intersections with f in the interval $[1, 3]$ if $k =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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19. Suppose f is continuous on the closed interval $[0,4]$ and suppose $f(0) = 1, f(1) = 2, f(2) = 0, f(3) = -3,$ $f(4) = 3$. Which of the following statements about the zeros of f on $[0,4]$ is always true?

B

(A) f has exactly one zero on $[0, 4]$. False, could be more

(B) f has more than one zero on $[0, 4]$.

(C) f has more than two zeros on $[0, 4]$. Could be exactly two.

(D) f has exactly two zeros on $[0, 4]$. False, could be more