Calculus

1.5 Algebraic Properties of Limits and Piecewise Functions

Write your questions and thoughts here!

Notes

$$x + x =$$

$$\lim_{x \to c}[f(x) + f(x)] =$$

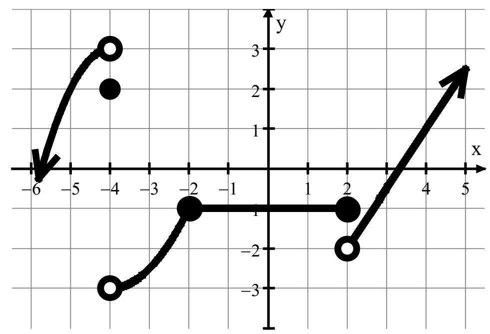
Example 1:

$\lim_{x \to -1} f(x) = 2$	$\lim_{x \to 1} f(x) = 4$	$\lim_{x \to 1} g(x) = 6$

The table above gives selected limits of the functions f and g. What is $\lim_{x \to 1}\left(f(-x) + \frac{g(x)}{2}\right)$?

Example 2:

The graph of the function f is shown on the right. What is $\lim_{x \to 4} f\big(f(x)\big)$?



Graph of f

Example 3:

$f(5) = 1$	$\lim_{x \to 5} f(x) = 6$
$g(5) = 2$	$\lim_{x \to 5} g(x) = -1$
$h(5) = 3$	$\lim_{x \to 5} h(x) = 5$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim_{x \to 5}\Big(h(x)\big(f(x) + 2g(x)\big)\Big) - h(5)$?

Example 4: Piecewise Functions

Piecewise defined functions and limits

$$f(x) = \begin{cases} \sqrt{11 - x}, & x < -5 \\ \dfrac{x + 3}{5 - x^2}, & x \geq -5 \end{cases}$$

a. $\lim_{x \to -5^-} f(x) =$

b. $\lim_{x \to -5^+} f(x) =$

c. $\lim_{x \to -5} f(x) =$

$$g(x) = \begin{cases} \sqrt{10 - x^2}, & x < -1 \\ \dfrac{26 - 5x^2}{7} & -1 < x \leq e \\ \ln x^3, & x > e \end{cases}$$

a. $\lim_{x \to -1} g(x) =$

b. $\lim_{x \to e^+} g(x) =$

c. $\lim_{x \to e} g(x) =$

1.5 Algebraic Properties of Limits

Calculus

Practice

Use the table for each problem to find the given limits.

1.

$\lim_{x\to3} f(x) = 4$	$\lim_{x\to-3} f(x) = 2$	$\lim_{x\to3} g(x) = 1$	$\lim_{x\to-3} g(x) = 5$

a. $\lim_{x\to3}\left(2f(x) + g(-x)\right)$

b. $\lim_{x\to-3}\left(\frac{g(x)}{f(-x)}\right)$

2.

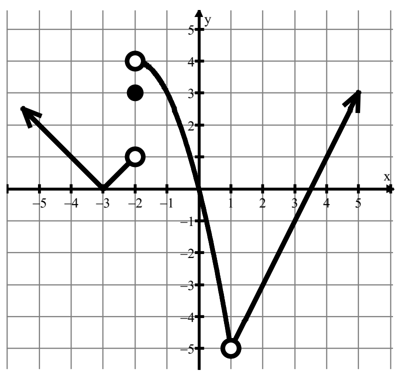
$\lim_{x\to2} f(x) = -1$	$\lim_{x\to1} f(x) = 6$	$\lim_{x\to4} f(x) = 2$	$\lim_{x\to-2} f(x) = -3$

a. $\lim_{x\to2}\left(f(2x) - f\left(\frac{x}{2}\right)\right)$

b. $\lim_{x\to2}\left(\frac{f\left(\frac{x}{2}\right)}{f(-x)}\right)$

Use the graph for each problem to find the given limits.

3.

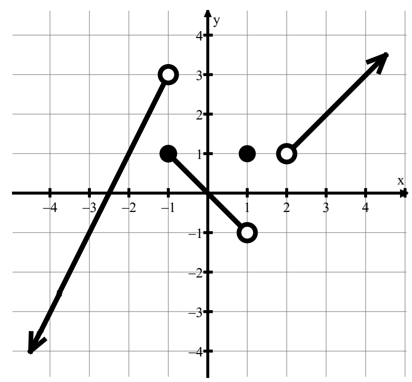


Graph of f

a. $\lim_{x\to3} f(f(x)) =$

b. $\lim_{x\to1} f(f(x)) =$

4.



Graph of f

a. $\lim_{x\to-2} f(f(x)) =$

b. $\lim_{x\to4} f(f(x)) =$

Use the table for each problem to find the given limits.

5.

$f(1) = 4$	$\lim_{x\to1} f(x) = -1$
$g(1) = -2$	$\lim_{x\to1} g(x) = 3$
$h(1) = -3$	$\lim_{x\to1} h(x) = 6$

a. $\lim_{x\to1}\left(\left(f(x)\right)^2 - h(x)\right) - g(1)$

b. $f(1) + \lim_{x\to1}\left(-g(x)\right)$

6.

$f(-2) = 7$	$\lim_{x \to -2} f(x) = 2$
$g(-2) = 1$	$\lim_{x \to -2} g(x) = -1$
$h(-2) = -4$	$\lim_{x \to -2} h(x) = -3$

a. $\lim_{x \to -2} \left(h(x)\left(2f(x)\right)\right) + h(-2)$

b. $f(-2) \lim_{x \to -2} \left(g(x) - h(x)\right)$

Use the piecewise functions to find the given values.

7. $g(x) = \begin{cases} \sqrt{5 - x}, & x < -4 \\ x^2 - 5, & -4 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$

a. $\lim_{x \to 2^-} g(x) =$

b. $\lim_{x \to -4^+} g(x) =$

c. $g(2) =$

d. $\lim_{x \to -4^-} g(x) =$

e. $\lim_{x \to 2^+} g(x) =$

f. $\lim_{x \to 2} g(x) =$

g. $\lim_{x \to -4} g(x) =$

h. $g(-4) =$

8. $h(x) = \begin{cases} -|x|, & x \le -5 \\ 20 - x^2, & -5 < x \le 3 \\ 4x - 1, & x > 3 \end{cases}$

a. $\lim_{x \to -5^+} h(x) =$

b. $\lim_{x \to -5} h(x) =$

c. $h(3) =$

d. $\lim_{x \to -5^-} h(x) =$

e. $\lim_{x \to 3^+} h(x) =$

f. $\lim_{x \to 3} h(x) =$

g. $h(-5) =$

h. $\lim_{x \to 3^-} h(x) =$

9. $w(\theta) = \begin{cases} \sin\theta, & \theta \le \pi \\ \cos\theta, & \pi < \theta < 2\pi \\ \tan\theta, & \theta > 2\pi \end{cases}$

a. $\lim_{\theta \to \pi^-} w(\theta) =$

b. $w(\pi) =$

c. $\lim_{\theta \to \pi^+} w(\theta) =$

d. $\lim_{\theta \to 2\pi^-} w(\theta) =$

e. $\lim_{\theta \to \pi} w(\theta) =$

f. $\lim_{\theta \to 2\pi^+} w(\theta) =$

g. $\lim_{\theta \to 2\pi} w(\theta) =$

h. $w(2\pi) =$

10. $f(x) = \begin{cases} \frac{1}{x+6}, & x < -2 \\ 2^x, & -2 \le x < 0 \\ x^2 - 4, & x \ge 0 \end{cases}$

a. $\lim_{x \to -2} f(x) =$

b. $\lim_{x \to -2^-} f(x) =$

c. $\lim_{x \to -2^+} f(x) =$

d. $\lim_{x \to 0} f(x) =$

e. $\lim_{x \to 0^-} f(x) =$

f. $\lim_{x \to 0^+} f(x) =$

g. $f(-2) =$

h. $f(0) =$

11. If f is a continuous function such that $f(3) = 7$, which of the following statements must be true?

(A) $\lim_{x\to 3} f(3x) = 9$ (B) $\lim_{x\to 3} f(2x) = 14$ (C) $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = 7$

(D) $\lim_{x\to 3} f(x^2) = 49$ (E) $\lim_{x\to 3} \left(f(x)\right)^2 = 49$

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12. If $f(x) = \begin{cases} \ln 3x, & 0 < x \le 3 \\ x\ln 3, & 3 < x \le 4 \end{cases}$, then $\lim_{x\to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3\ln 3$ (D) $3 + \ln 3$ (E) nonexistent

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13. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent