

# 1.5 Algebraic Properties of Limits

Calculus

Solutions

Practice

Use the table for each problem to find the given limits.

1.

$\lim_{x \rightarrow 3} f(x) = 4$	$\lim_{x \rightarrow -3} f(x) = 2$	$\lim_{x \rightarrow 3} g(x) = 1$	$\lim_{x \rightarrow -3} g(x) = 5$
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a.  $\lim_{x \rightarrow 3} (2f(x) + g(-x))$   
 $2(4) + 5$   
 $13$

b.  $\lim_{x \rightarrow -3} \left( \frac{g(x)}{f(-x)} \right) = \frac{5}{4}$

2.

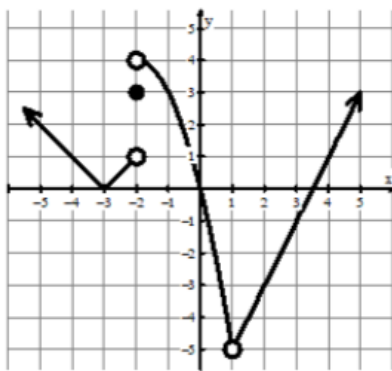
$\lim_{x \rightarrow 2} f(x) = -1$	$\lim_{x \rightarrow 1} f(x) = 6$	$\lim_{x \rightarrow 4} f(x) = 2$	$\lim_{x \rightarrow -2} f(x) = -3$
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a.  $\lim_{x \rightarrow 2} \left( f(2x) - f\left(\frac{x}{2}\right) \right)$   
 $2 - 6$   
 $-4$

b.  $\lim_{x \rightarrow 2} \left( \frac{f\left(\frac{x}{2}\right)}{f(-x)} \right) = \frac{6}{-3} = -2$

Use the graph for each problem to find the given limits.

3.

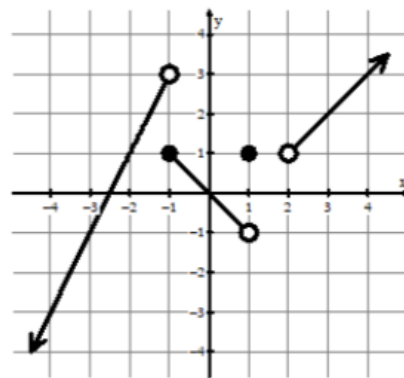


Graph of  $f$

a.  $\lim_{x \rightarrow 3} f(f(x)) = \lim_{x \rightarrow -1} f(x) = 3$

b.  $\lim_{x \rightarrow 1} f(f(x)) = \lim_{x \rightarrow -5} f(x) = 2$

4.



Graph of  $f$

a.  $\lim_{x \rightarrow -2} f(f(x)) = \lim_{x \rightarrow 1} f(x) = \text{Does not exist}$

b.  $\lim_{x \rightarrow 4} f(f(x)) = \lim_{x \rightarrow 3} f(x) = 2$

Use the table for each problem to find the given limits.

5.

$f(1) = 4$	$\lim_{x \rightarrow 1} f(x) = -1$
$g(1) = -2$	$\lim_{x \rightarrow 1} g(x) = 3$
$h(1) = -3$	$\lim_{x \rightarrow 1} h(x) = 6$

a.  $\lim_{x \rightarrow 1} \left( (f(x))^2 - h(x) \right) - g(1)$   
 $(-1)^2 - 6 - (-2)$   
 $-5 + 2$   
 $-3$

b.  $f(1) + \lim_{x \rightarrow 1} (-g(x))$   
 $4 + (-3)$   
 $4 - 3$   
 $1$

6.

$f(-2) = 7$	$\lim_{x \rightarrow -2} f(x) = 2$
$g(-2) = 1$	$\lim_{x \rightarrow -2} g(x) = -1$
$h(-2) = -4$	$\lim_{x \rightarrow -2} h(x) = -3$

$$\text{a. } \lim_{x \rightarrow -2} (h(x)(2f(x))) + h(-2)$$

$$((-3)(2 \cdot 7)) + (-4)$$

$$-12 - 4 = \boxed{-16}$$

$$\text{b. } f(-2) \lim_{x \rightarrow -2} (g(x) - h(x))$$

$$7 \cdot (-1 - (-3))$$

$$7(2) = \boxed{14}$$

Use the piecewise functions to find the given values.

$$7. g(x) = \begin{cases} \sqrt{5-x}, & x < -4 \\ x^2 - 5, & -4 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases}$$

$$\text{a. } \lim_{x \rightarrow 2^-} g(x) =$$

$$(2)^2 - 5 = \boxed{-1}$$

$$\text{b. } \lim_{x \rightarrow -4^+} g(x) =$$

$$(-4)^2 - 5 = \boxed{11}$$

$$\text{c. } g(2) =$$

$$(2) - 3 = \boxed{-1}$$

$$\text{d. } \lim_{x \rightarrow -4^-} g(x) =$$

$$\sqrt{5 - (-4)} = \sqrt{9} = \boxed{3}$$

$$\text{e. } \lim_{x \rightarrow 2^+} g(x) =$$

$$(2) - 3 = \boxed{-1}$$

$$\text{f. } \lim_{x \rightarrow 2} g(x) =$$

$$\boxed{-1}$$

$$\text{g. } \lim_{x \rightarrow -4} g(x) =$$

Does not exist

$$\text{h. } g(-4) =$$

$$(-4)^2 - 5 = \boxed{11}$$

$$8. h(x) = \begin{cases} -|x|, & x \leq -5 \\ 20 - x^2, & -5 < x \leq 3 \\ 4x - 1, & x > 3 \end{cases}$$

$$\text{a. } \lim_{x \rightarrow -5^+} h(x) =$$

$$20 - (-5)^2 = \boxed{-5}$$

$$\text{b. } \lim_{x \rightarrow -5} h(x) =$$

$$\boxed{-5}$$

$$\text{c. } h(3) =$$

$$20 - (3)^2 = \boxed{11}$$

$$\text{d. } \lim_{x \rightarrow -5^-} h(x) =$$

$$-|-5| = \boxed{-5}$$

$$\text{e. } \lim_{x \rightarrow 3^+} h(x) =$$

$$4(3) - 1 = \boxed{11}$$

$$\text{f. } \lim_{x \rightarrow 3} h(x) =$$

$$\boxed{11}$$

$$\text{g. } h(-5) =$$

$$-|-5| = \boxed{-5}$$

$$\text{h. } \lim_{x \rightarrow 3^-} h(x) =$$

$$20 - (3)^2 = \boxed{11}$$

$$9. w(\theta) = \begin{cases} \sin \theta, & \theta \leq \pi \\ \cos \theta, & \pi < \theta < 2\pi \\ \tan \theta, & \theta > 2\pi \end{cases}$$

$$\text{a. } \lim_{\theta \rightarrow \pi^-} w(\theta) =$$

$$\sin(\pi) = \boxed{0}$$

$$\text{b. } w(\pi) =$$

$$\sin(\pi) = \boxed{0}$$

$$\text{c. } \lim_{\theta \rightarrow \pi^+} w(\theta) =$$

$$\cos(\pi) = \boxed{-1}$$

$$\text{d. } \lim_{\theta \rightarrow 2\pi^-} w(\theta) =$$

$$\cos(2\pi) = \boxed{1}$$

$$\text{e. } \lim_{\theta \rightarrow \pi} w(\theta) =$$

Does not exist

$$\text{f. } \lim_{\theta \rightarrow 2\pi^+} w(\theta) =$$

$$\tan(2\pi) = \boxed{0}$$

$$\text{g. } \lim_{\theta \rightarrow 2\pi} w(\theta) =$$

Does not exist

$$\text{h. } w(2\pi) =$$

Does not exist

$$10. f(x) = \begin{cases} \frac{1}{x+6}, & x < -2 \\ 2^x, & -2 \leq x < 0 \\ x^2 - 4, & x \geq 0 \end{cases}$$

$$\text{a. } \lim_{x \rightarrow -2} f(x) =$$

$$\boxed{\frac{1}{4}}$$

$$\text{b. } \lim_{x \rightarrow -2^-} f(x) =$$

$$\frac{1}{-2+6} = \boxed{\frac{1}{4}}$$

$$\text{c. } \lim_{x \rightarrow -2^+} f(x) =$$

$$2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$$

$$\text{d. } \lim_{x \rightarrow 0} f(x) =$$

Does not exist

$$\text{e. } \lim_{x \rightarrow 0^-} f(x) =$$

$$2^0 = \boxed{1}$$

$$\text{f. } \lim_{x \rightarrow 0^+} f(x) =$$

$$0 - 4 = \boxed{-4}$$

$$\text{g. } f(-2) =$$

$$2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$$

$$\text{h. } f(0) =$$

$$0^2 - 4 = \boxed{-4}$$

1.5 Algebraic Properties of Limits

11. If  $f$  is a continuous function such that  $f(3) = 7$ , which of the following statements must be true?

- (A)  $\lim_{x \rightarrow 3} f(3x) = 9$       (B)  $\lim_{x \rightarrow 3} f(2x) = 14$       (C)  $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = 7$   
 (D)  $\lim_{x \rightarrow 3} f(x^2) = 49$       (E)  $\lim_{x \rightarrow 3} (f(x))^2 = 49$       E

12. If  $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$  then  $\lim_{x \rightarrow 3} f(x)$  is

- $\ln 3^x$        $\ln 27 = \ln 9?$       E  
 (A)  $\ln 9$       (B)  $\ln 27$       (C)  $3 \ln 3$       (D)  $3 + \ln 3$       (E) nonexistent

13. If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4 \end{cases}$  then  $\lim_{x \rightarrow 2} f(x)$  is

- $\ln 2 = 2^2 \ln 2$       E  
 $\ln 2 = 4 \cdot \ln 2$       False!  
 (A)  $\ln 2$       (B)  $\ln 8$       (C)  $\ln 16$       (D) 4      (E) nonexistent