

1.6 Algebraic Manipulation of Limits

Calculus

Solutions

Practice

Evaluate each limit.

<p>1. $\lim_{x \rightarrow 2} (x - x^2)$</p> $2 - 2^2$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">-2</div>	<p>2. $\lim_{x \rightarrow 5} (x + 1)^2$</p> $(5 + 1)^2$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">36</div>	<p>3. $\lim_{x \rightarrow 1} \frac{x^2 - 5x}{x - 1} = \frac{x(x - 5)}{x - 1}$</p> <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">Does not exist</div>	<p>4. $\lim_{x \rightarrow 1} \frac{x^2 + x - 30}{x - 1}$</p> $\frac{(x + 6)(x - 5)}{x - 1}$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">Does not exist</div>
<p>5. $\lim_{x \rightarrow 0} \frac{3x}{\sin x}$</p> <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">3</div>	<p>6. $\lim_{x \rightarrow 0} \frac{\sin(2x)}{3x}$</p> <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">$\frac{2}{3}$</div>	<p>7. $\lim_{x \rightarrow -2} (3x^2 - x + 1)$</p> $3(-2)^2 - (-2) + 1$ $12 + 2 + 1$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">15</div>	<p>8. $\lim_{x \rightarrow 3} (2x^2 + 5x - 6)$</p> $2(3)^2 + 5(3) - 6$ $18 + 15 - 6$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">27</div>
<p>9. $\lim_{x \rightarrow -7} \frac{2x^3 + 11x^2 - 21x}{x^2 + 7x}$</p> $\frac{x(2x^2 + 11x - 21)}{x(x + 7)}$ $\frac{x(x + 7)(2x - 3)}{x(x + 7)}$ $\lim_{x \rightarrow -7} (2x - 3)$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">-17</div>	<p>10. $\lim_{x \rightarrow 8} \frac{x^2 + 2x - 80}{x - 8}$</p> $\frac{(x - 8)(x + 10)}{x - 8}$ $\lim_{x \rightarrow 8} (x + 10)$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">18</div>	<p>11. $\lim_{x \rightarrow \frac{1}{3}} \frac{6x^2 + 13x - 5}{3x - 1}$</p> $\frac{(3x - 1)(2x + 5)}{3x - 1}$ $\lim_{x \rightarrow \frac{1}{3}} (2x + 5)$ $\frac{2}{3} + 5$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">$\frac{17}{3}$</div>	<p>12. $\lim_{x \rightarrow 0} \frac{7x^2 + x}{x}$</p> $\frac{x(7x + 1)}{x}$ $\lim_{x \rightarrow 0} (7x + 1)$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">1</div>
<p>13. $\lim_{x \rightarrow -3} 14$</p> <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">14</div>	<p>14. $\lim_{x \rightarrow 0} \frac{x^2 + 2x - 8}{x - 4}$</p> $\frac{0^2 + 2(0) - 8}{0 - 4}$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">2</div>	<p>15. $\lim_{x \rightarrow -2} \frac{x^2 - 4x - 10}{x}$</p> $\frac{(-2)^2 - 4(-2) - 10}{-2}$ $\frac{4 + 8 - 10}{-2}$ $\frac{2}{-2}$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">-1</div>	<p>16. $\lim_{x \rightarrow 0} \frac{3x^2 + 5x}{x}$</p> $\frac{x(3x + 5)}{x}$ $\lim_{x \rightarrow 0} (3x + 5)$ <div style="border: 1px solid red; padding: 5px; width: fit-content; margin: 10px auto;">5</div>

$$17. \lim_{x \rightarrow 4} \frac{5x^2 - 21x + 4}{x - 4}$$

$$\frac{(x-4)(5x-1)}{x-4}$$

$$\lim_{x \rightarrow 4} (5x-1)$$

$$20-1$$

19

$$18. \lim_{x \rightarrow \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$$

$$\frac{-(2x^2+x-1)}{2x-1}$$

$$\frac{-(2x-1)(x+1)}{2x-1}$$

$$\lim_{x \rightarrow \frac{1}{2}} (-x-1)$$

$-\frac{3}{2}$

$$19. \lim_{x \rightarrow \pi} \cos x$$

$$\cos \pi$$

-1

$$20. \lim_{x \rightarrow \frac{\pi}{8}} \sin(4x)$$

$$\sin\left(4 \cdot \frac{\pi}{8}\right)$$

1

$$21. \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{2-x}$$

$$\frac{(x-2)(x+8)}{-(x-2)}$$

$$\lim_{x \rightarrow 2} (-x-8)$$

-10

$$22. \lim_{x \rightarrow 5} \frac{2x^2 - 17x + 35}{5-x}$$

$$\frac{(x-5)(2x-7)}{-(x-5)}$$

$$\lim_{x \rightarrow 5} (-2x+7)$$

$$-10+7$$

-3

$$23. \lim_{x \rightarrow 0} \frac{(1-\cos^2 x) \sin x}{x^2}$$

$$\frac{(1-\cos x)(1+\cos x) \sin x}{x \cdot x}$$

$$\downarrow \quad \quad \quad \downarrow$$

$$0 \cdot (1+\cos x) \cdot 1$$

0

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Test Prep

24. Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{3x}$ is

$$\frac{\ln(1)}{3} = \frac{0}{3}$$

- (A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

25. $\lim_{x \rightarrow 0} 4 \frac{\sin x \cos x - \sin x}{x^2}$ is

$$\frac{\sin x (\cos x - 1)}{x \cdot x} = 1 \cdot 0$$

- (A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

26. $\lim_{x \rightarrow a} \frac{x^2 - 2ax + a^2}{x - a} =$

$$\frac{(x-a)(x-a)}{x-a} = \lim_{x \rightarrow a} (x-a) = a-a$$

- (A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

27. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 5\cos x - 5}{2x} \right) =$

$$\frac{3x^2}{2x} + \frac{5(\cos x - 1)}{2x} = \frac{3x}{2} + \frac{5}{2} \left(\frac{\cos x - 1}{x} \right)$$

- (A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist