## 10.1 Convergent and Divergent Infinite Series

Calculus

**Practice**

1. Given the infinite series $\sum_{n=1}^{\infty}(-1)^n$, find the sequence of partial sums $S_1, S_2, S_3, S_4$, and $S_5$.

$S_1 = -1$ $\quad S_3 = S_2 - 1 = -1$ $\quad S_5 = S_4 - 1 = -1$

$S_2 = S_1 + 1 = 0$ $\quad S_4 = S_3 + 1 = 0$

2. Find the sequence of partial sums $S_1, S_2, S_3, S_4$, and $S_5$ for the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \cdots$.

$S_1 = 1$ $\quad S_3 = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$ $\quad S_5 = \frac{23}{12} + \frac{1}{8} = \frac{49}{24}$

$S_2 = \frac{3}{2}$ $\quad S_4 = \frac{7}{4} + \frac{1}{6} = \frac{23}{12}$

3. If the infinite series $\sum_{n=1}^{\infty} a^n$ has $n$th partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$, what is the sum of the series?

$\lim_{n\to\infty} S_n$ $\boxed{\text{diverges}}$ $\quad -1, 1, -1, 1, -1, \ldots$

4. The infinite series $\sum_{n=1}^{\infty} a^n$ has $n$th partial sum $S_n = \frac{n}{4n+1}$ for $n \geq 1$. What is the sum of the series?

$\lim_{n\to\infty} \frac{n}{4n+1} = \frac{1}{4}$

5. Use a calculator to find the partial sum $S_n$ of the series $\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$ for $n = 100, 500, 1000$.

$S_{100} \simeq 3.6078$ $\quad S_{500} \simeq 3.6547$ $\quad S_{1000} \simeq 3.6606$

6. Show that the sequence with the given $n$th term $a_n = 1 + 2n$ is monotonic.

$a_1 = 3$ $\quad a_2 = 5$ $\quad a_3 = 7$ $\quad a_4 = 9$ $\quad \ldots$

$a_n$ is monotonic because $a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_n$

7. What is the $n$th partial sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{2^{n+1}}$?

$S_n = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots \frac{1}{2^{n+1}}$

$\frac{1}{2}S_n = \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots \frac{1}{2^{n+2}}$

$S_n - \frac{1}{2}S_n = \frac{1}{2^2} - \frac{1}{2^{n+2}}$

$\frac{1}{2}S_n = \frac{1}{2^2} - \frac{1}{2^{n+2}}$

$S_n = 2\left[\frac{1}{2^2} - \frac{1}{2^{n+2}}\right]$

$\boxed{S_n = \frac{1}{2} - \frac{1}{2^{n+1}}}$

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8. Which of the following could be the $n$th partial sum for the infinite series $\sum_{n=1}^{\infty} \frac{1}{4^n}$?

$S_n = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n}$

$\frac{1}{4} S_n = \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots \frac{1}{4^{n+1}}$

$S_n - \frac{1}{4} S_n = \frac{1}{4} - \frac{1}{4^{n+1}}$

$\frac{3}{4} S_n = \frac{1}{4} - \frac{1}{4^{n+1}}$

$3 S_n = 1 - \frac{1}{4^n}$

$S_n = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$

(A) $S_n = \frac{1}{3}\left(1 + \frac{1}{4^n}\right)$ (B) $S_n = \frac{1}{3}\left(1 - \frac{1}{4^{n+1}}\right)$ (C) $S_n = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$ (D) $S_n = \frac{1}{4}\left(1 - \frac{1}{3^n}\right)$

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9. If the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent and has a sum of $\frac{7}{8}$, which of the following could be the $n$th partial sum?

$\lim_{n\to\infty} S_n = \frac{7}{8}$

(A) $S_n = \frac{7n+1}{8n^2+1} \rightarrow 0$

(B) $S_n = \frac{7n^2+1}{8n+1} \rightarrow \infty$

(C) $S_n = 2\left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right) \rightarrow \frac{14}{8}$

(D) $S_n = \left(\frac{7}{8} - \frac{1}{n+2} - \frac{1}{n+3}\right) \rightarrow \frac{7}{8}$

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10. Which of the following sequences with the given $n$th term is bounded and monotonic?

Bounded, not monotonic ↓ (A) $a_n = 2 + (-1)^n$

not bounded, monotonic ↓ (B) $a_n = \frac{n^2}{n+1}$

bounded, monotonic ↓ (C) $a_n = \frac{3n}{n+2}$

bounded, not monotonic (D) $a_n = \frac{\cos n}{n}$