

2.10 Derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$

Calculus

Solutions

Practice

Find the derivative of each function.

1. $y = 5 - \csc x$

$$y' = \csc x \cot x$$

2. $h(x) = 2x \tan(x)$

$$h'(x) = 2 \tan x + 2x \sec^2 x$$

3. $r = \frac{\sin \theta}{\theta}$

$$\frac{dr}{d\theta} = \frac{\cos \theta \cdot \theta - \sin \theta}{\theta^2}$$

4. $g(x) = \frac{\cot x}{x}$

$$g'(x) = \frac{-\csc^2 x \cdot x - \cot x}{x^2}$$

5. $f(x) = \frac{1}{2 \cos x} = \frac{1}{2} \sec x$

$$f'(x) = \frac{1}{2} \sec x \tan x$$

6. $y = 5x \sec x$

$$\frac{dy}{dx} = 5 \sec x + 5x \sec x \tan x$$

Find the derivative at the given x -value. Show your work!

7. $f(x) = 3 \tan x$ at $x = \frac{2\pi}{3}$.

$$f'(x) = 3 \sec^2 x$$

$$f'(x) = \frac{3}{\cos^2 x}$$

$$f'\left(\frac{2\pi}{3}\right) = \frac{3}{\left(-\frac{1}{2}\right)^2}$$

$$= 3 \cdot 4$$

$$= \boxed{12}$$

8. $f(x) = 2 \sec x$ at $x = \frac{\pi}{4}$.

$$f'(x) = 2 \sec x \tan x$$

$$f'(x) = \frac{2}{\cos x} \tan x$$

$$f'\left(\frac{\pi}{4}\right) = \frac{2}{\frac{\sqrt{2}}{2}} \cdot (1)$$

$$= 2 \cdot \frac{2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}}$$

$$= \boxed{2\sqrt{2}}$$

9. $f(x) = x \cot x$ at $x = \frac{\pi}{6}$.

$$f'(x) = 1 \cdot \cot x + x(-\csc^2 x)$$

$$f'(x) = \frac{\cos x}{\sin x} - \frac{x}{\sin^2 x}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} - \frac{\frac{\pi}{6}}{\left(\frac{1}{2}\right)^2}$$

$$= \sqrt{3} - \frac{\pi}{6} \cdot 4$$

$$= \boxed{\sqrt{3} - \frac{2\pi}{3}}$$

Estimate the derivative at the given x -value by using a calculator.

10. $f(x) = \sin^2\left(\frac{x}{5}\right)$ at $x = 1.8$.

$$f'(1.8) = 0.1318$$

11. $f(x) = \frac{\cot(x^2)}{2}$ at $x = -1$.

$$f'(-1) = 1.412$$

12. $f(x) = 3 \sec(e^x)$ at $x = 2.5$.

$$f'(2.5) = -15.9228$$

Find the equations of both the normal line and the tangent line.

13. $y = \sec x$ at $x = \pi$

$$y(\pi) = \frac{1}{\cos(\pi)} = -1 \leftarrow y_1$$

$$y' = \sec x \tan x$$

$$y' = \frac{1}{\cos \pi} \cdot \frac{\sin \pi}{\cos \pi} = 0 \leftarrow m$$

$$y + 1 = 0(x - \pi)$$

Tangent: $y = -1$

Normal: $x = \pi$

14. $y = \tan x$ at $x = \frac{\pi}{3}$

$$y\left(\frac{\pi}{3}\right) = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$y' = \sec^2 x$$

$$y'\left(\frac{\pi}{3}\right) = \frac{1}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

Tangent: $y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

Normal: $y - \sqrt{3} = -\frac{1}{4}\left(x - \frac{\pi}{3}\right)$

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Test Prep

Evaluate each limit.

15. $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{3}+h\right) - \tan\left(\frac{\pi}{3}\right)}{h} =$ Def. of Der.

$$f(x) = \tan x$$

$$f'(x) = \sec^2 x$$

$$f'\left(\frac{\pi}{3}\right) = \sec^2\left(\frac{\pi}{3}\right) = 4$$

Same as problem # 14

16. $\lim_{h \rightarrow 0} \frac{\sec\left(\frac{\pi}{6}+h\right) - \sec\left(\frac{\pi}{6}\right)}{h} =$

$$f(x) = \sec x$$

$$f'(x) = \sec x \tan x$$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{6}\right)} \cdot \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \cdot \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{4}{3}$$

$$\frac{2}{3}$$