

2.4 Differentiability and Continuity

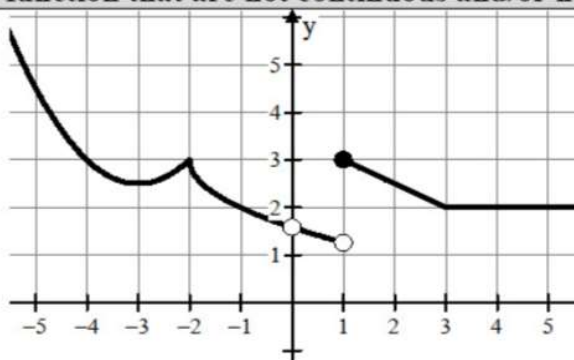
Calculus

Solutions

Practice

Identify any x -values of the function that are not continuous and/or not differentiable.

1.



x -values where the function is not continuous.

$$x = 0$$

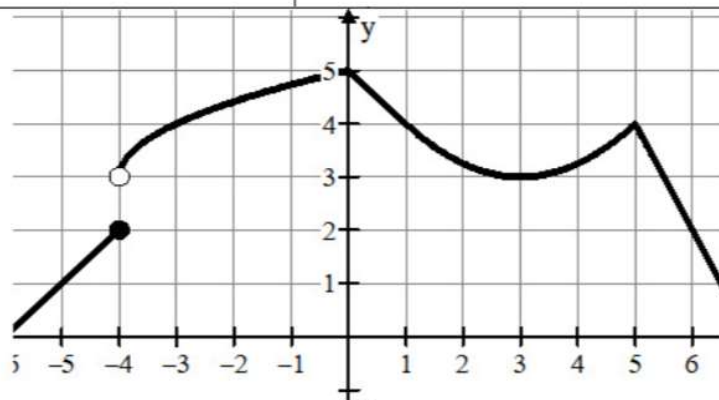
$$x = 1$$

x -values where the function is continuous, but not differentiable.

$$x = -2$$

$$x = 3$$

2.



x -values where the function is not continuous.

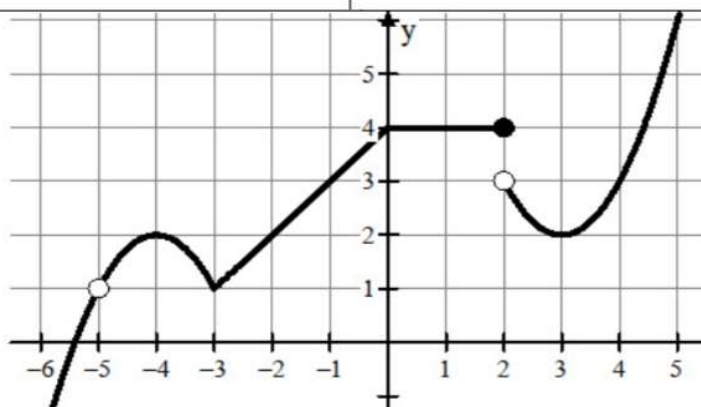
$$x = -4$$

x -values where the function is continuous, differentiable.

$$x = 0$$

$$x = 5$$

3.



x -values where the function is not continuous.

$$x = -5$$

$$x = 2$$

x -values where the function is continuous, differentiable.

$$x = -3$$

$$x = 0$$

2.4 Differentiability and Continuity

4. f is continuous for $a \leq x \leq b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f .

(B) $\lim_{x \rightarrow c} f(x) \neq f(c)$

(C) The graph of f has a cusp at $x = c$.

(D) $f(c)$ is undefined.

(E) None of the above

5. If g is differentiable at $x = c$, which of the following must be true?

I. g is continuous at $x = c$.

II. $\lim_{x \rightarrow c} g(x)$ exists.

III. $\lim_{x \rightarrow c} \frac{g(x) - g(c)}{x - c}$ exists.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

6. Let h be the function given by $h(x) = |x - 4|$. Which of the following statements about h are true?

I. h is continuous at $x = 4$.

II. h is differentiable at $x = 4$.

III. h has an absolute minimum at $x = 4$.

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

7. If f is a differentiable function such that $f(2) = 5$ and $f'(2) = 7$, which of the following statements could be false?

(A) $\lim_{x \rightarrow 2} f(x) = 5$

(B) $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

(C) $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 7$

(D) $\lim_{h \rightarrow 0} \frac{f(2+h) - 5}{h} = 7$

(E) $\lim_{h \rightarrow 0} f'(x) = 7$