3.2 Implicit Differentiation

Calculus

Solutions **Practice**

Find $\frac{dy}{dx}$.

1. $5x^2 + 2y^3 = 4$

$$10x + 6y^2\frac{dy}{dx} = 0$$

$$6y^2\frac{dy}{dx} = -10x$$

$$\boxed{\frac{dy}{dx} = -\frac{5x}{3y^2}}$$

2. $5y^2 + 3 = x^2$

$$10y\frac{dy}{dx} = 2x$$

$$\boxed{\frac{dy}{dx} = \frac{x}{5y}}$$

3. $\sin(x + y) = 2x$

$$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 2$$

$$1+\frac{dy}{dx} = \frac{2}{\cos(x+y)}$$

$$\boxed{\frac{dy}{dx} = 2\sec(x+y) - 1}$$

4. $4x + 1 = \cos y^2$

$$4 = -\sin(y^2)\cdot 2y\frac{dy}{dx}$$

$$\frac{-4}{\sin(y^2)} = 2y\frac{dy}{dx}$$

$$\boxed{\frac{-2\csc(y^2)}{y} = \frac{dy}{dx}}$$

5. $5x^2 - e^{4y^2} = -6$

$$10x - e^{4y^2}\cdot\left(8y\frac{dy}{dx}\right) = 0$$

$$-e^{4y^2}\left(8y\frac{dy}{dx}\right) = -10x$$

$$\boxed{\frac{dy}{dx} = \frac{5x}{4ye^{4y^2}}}$$

6. $\ln(y^3) = 5x + 3$

$$\frac{3}{y}\frac{dy}{dx} = 5$$

$$\boxed{\frac{dy}{dx} = \frac{5y}{3}}$$

7. $x^2 = 4y^3 + 5y^2$

$$\frac{2x}{2} = \frac{12y^2y' + 10yy'}{2}$$

$$x = y'(6y^2 + 5y)$$

$$\frac{dy}{dx} = \boxed{\frac{x}{6y^2 + 5y}}$$

8. $5x^3 - 2y = 5y^3$

$$15x^2 - 2y' = 15y^2y'$$

$$15x^2 = 15y^2y' + 2y'$$

$$15x^2 = y'(15y^2 + 2)$$

$$\frac{dy}{dx} = \boxed{\frac{15x^2}{15y^2 + 2}}$$

9. $\ln y^2 + \cos^2 x = 1 - y$

$$\frac{2}{y}y' + 2\cos x(-\sin x) = -y'$$

$$\frac{2}{y}y' + y' = 2\cos x\sin x$$

$$y'\left(\frac{2}{y} + 1\right) = 2\cos x\sin x$$

$$\frac{dy}{dx} = \boxed{\frac{2\cos x\sin x}{\frac{2}{y} + 1}}$$

10. $\sin\left(\frac{y}{2}\right) + e^y = 4x$

$$\cos\left(\frac{y}{2}\right)\cdot\frac{1}{2}y' + e^y y' = 4$$

$$y'\left(\frac{1}{2}\cos\left(\frac{y}{2}\right) + e^y\right) = 4$$

$$\frac{dy}{dx} = \boxed{\frac{4}{\frac{1}{2}\cos\left(\frac{y}{2}\right) + e^y}}$$

11. $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2y' = 6y + 6xy'$$

$$\frac{3y^2y' - 6xy'}{3} = \frac{6y - 3x^2}{3}$$

$$y'(y^2 - 2x) = 2y - x^2$$

$$\frac{dy}{dx} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$$

12. $\frac{x}{\sin y} = 5$

$$\frac{(1)\sin y - x\cos y\cdot y'}{\sin^2 y} = 0$$

$$\sin y - xy'\cos y = 0$$

$$-xy'\cos y = -\sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{x\cos y} = \boxed{\frac{\tan y}{x}}$$

13. $\ln x\, e^{3y} = 2y^2$

$$\frac{1}{x}e^{3y} + \ln x\, e^{3y}\cdot 3y' = 4yy'$$

$$3\ln x\, e^{3y}y' - 4yy' = -\frac{1}{x}e^{3y}$$

$$y'(3\ln x\, e^{3y} - 4y) = -\frac{e^{3y}}{x}$$

$$\frac{dy}{dx} = \boxed{-\frac{e^{3y}}{x(3\ln x\, e^{3y} - 4y)}}$$

Find the slope of the tangent line at the given x-value. Show work.

14. $2 = 3x^4 + xy^4$ at $(-1, 1)$

$0 = 12x^3 + (1)y^4 + x \cdot 4y^3 \frac{dy}{dx}$

$0 = 12(-1) + (1)^4 + (-1)(4)(1)^3 \frac{dy}{dx}$

$0 = -11 - 4\frac{dy}{dx}$

$\frac{dy}{dx} = \boxed{-11/4}$

15. $x \ln y = 4 - 2x$ at $(2, 1)$

$(1)\ln y + \frac{x}{y}\frac{dy}{dx} = -2$

$\ln(1) + \frac{2}{1}\frac{dy}{dx} = -2$

$0 + 2\frac{dy}{dx} = -2$

$\frac{dy}{dx} = \boxed{-1}$

Find the equation of the tangent line at the given x-value.

16. $x^2 + y^2 + 19 = 2x + 12y$ at $(4, 3)$

$2x + 2y\,y' = 2 + 12y'$

$2(4) + 2(3)\,y' = 2 + 12y'$

$8 + 6y' = 2 + 12y'$

$y' = \frac{-6}{-6} = 1$

$\boxed{y - 3 = 1(x - 4)}$

17. $x \sin 2y = y \cos 2x$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$(1)\sin 2y + x\cos(2y)\cdot 2y' = y'\cos(2x) + y(-\sin 2x)\cdot 2$

$\sin(\pi) + \frac{\pi}{4}\cos(\pi)\cdot 2y' = y'\cos(\frac{\pi}{2}) - \frac{\pi}{2}(\sin\frac{\pi}{2})\cdot 2$

$0 + \frac{\pi}{4}(-1)\cdot 2y' = y'(0) - \pi(1)$

$-\frac{\pi}{2}y' = -\pi$

$y' = 2$

$\boxed{y - \frac{\pi}{2} = 2(x - \frac{\pi}{4})}$

Find the equations of all horizontal and vertical tangent lines. Calculator allowed. Round to three decimals.

18. $x^2 + x + 2y^2 = 8$

$2x + 1 + 4y\,y' = 0$

$\frac{dy}{dx} = \frac{-2x-1}{4y}$

H.A when $-2x - 1 = 0$

$x = -\frac{1}{2}$

$(-\frac{1}{2})^2 + -\frac{1}{2} + 2y^2 = 8$

V.A. when $4y = 0$

$y = 0 \rightarrow x^2 + x = 8$

Horizontal: $y = \pm 2.031$

Vertical: $x = -3.372$ and 2.372

19. $x + y = y^2$

$1 + \frac{dy}{dx} = 2y\frac{dy}{dx}$

$1 = \frac{dy}{dx}(2y - 1)$

$\frac{dy}{dx} = \frac{1}{2y-1}$

V.A. when $2y - 1 = 0$

$y = \frac{1}{2}$

$x + \frac{1}{2} = (\frac{1}{2})^2$

Horizontal: None

Vertical: $x = -\frac{1}{4}$

3.2 Implicit Differentiation

Test Prep

20. Find the slope of the normal line to $y = x + \cos(xy)$ at $(0,1)$.

$$\frac{dy}{dx} = 1 - \sin(xy)\cdot\left(1\cdot y + x\frac{dy}{dx}\right)$$
$$\frac{dy}{dx} = 1 - \sin(0)\cdot\left(1\cdot 1 + 0\frac{dy}{dx}\right)$$
$$\frac{dy}{dx} = 1 - 0 = 1 \leftarrow$$ negative reciprocal

(A) 1 (B) -1 (C) 0 (D) 2 (E) Undefined

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21. The graph of $f(x)$, shown below, consists of a semicircle and two-line segments. $f'(1) =$

Circle: $x^2 + y^2 = r^2$

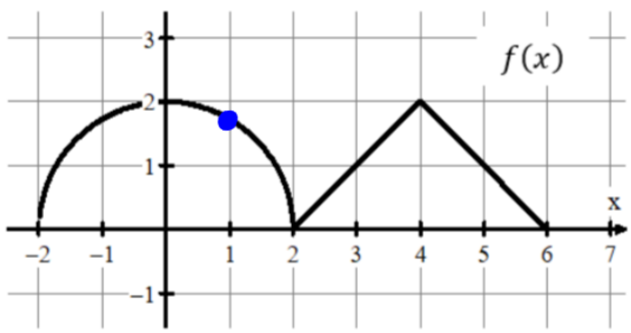
$x^2 + y^2 = 4$

$1^2 + y^2 = 4$

$y^2 = 3$

$y = \pm\sqrt{3}$

$f(1) = \sqrt{3}$



$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

$= -\frac{1}{\sqrt{3}}$

(A) -1 (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1 (E) $\sqrt{3}$

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22. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 1$.

$x^2(1) + 1^2 = 5$

$x^2 = 4$

$x = \pm 2$

Plug in

$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x + \frac{dy}{dx}(x^2 + 2) = 0$

$\frac{dy}{dx} = \frac{-2x}{x^2+2}$

Plug in

(A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm\frac{2}{3}$ (E) $\pm\frac{3}{2}$