3.4 Differentiating Inverse Trig Functions

Calculus

Solutions **Practice**

Find the derivative of each expression.

1. $\frac{d}{dx}\sin^{-1}(5x)$

$$\boxed{\frac{5}{\sqrt{1-25x^2}}}$$

2. $\frac{d}{dx}\csc^{-1}(4x^5)$

$$-\frac{20x^4}{|4x^5|\sqrt{16x^{10}-1}}$$

$$\boxed{-\frac{5}{|x|\sqrt{16x^{10}-1}}}$$

3. $\frac{d}{dx}\arctan(2x)$

$$\boxed{\frac{2}{4x^2+1}}$$

4. $\frac{d}{dx}\sec^{-1}(x^3)$

$$\frac{3x^2}{|x^3|\sqrt{x^6-1}}$$

$$\boxed{\frac{3}{|x|\sqrt{x^6-1}}}$$

5. $\frac{d}{dx}\csc 6x$

not an inverse!

$$\boxed{-6\csc(6x)\cot(6x)}$$

6. $\frac{d}{dx}\arccos(3x^2)$

$$\boxed{-\frac{6x}{\sqrt{1-9x^4}}}$$

7. $\frac{d}{dx}\cot^{-1}(-x)$

$$-\frac{-1}{x^2+1}$$

$$\boxed{\frac{1}{x^2+1}}$$

8. $\frac{d}{dx}\cos^{-1}(-7x)$

$$-\frac{-7}{\sqrt{1-49x^2}}$$

$$\boxed{\frac{7}{\sqrt{1-49x^2}}}$$

9. $\frac{d}{dx}\operatorname{arccsc}(x^6)$

$$-\frac{6x^5}{|x^6|\sqrt{x^{12}-1}}$$

$$\boxed{-\frac{6}{x\sqrt{x^{12}-1}}}$$

10. $\frac{d}{dx}\cot^{-1}(4x^4)$

$$\boxed{-\frac{16x^3}{16x^8+1}}$$

Find the tangent line equation of the curve at the given point.

11. $y = \arcsin(x)$ at the point where $x = \frac{\sqrt{2}}{2}$

$$y_1 = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$m = \frac{1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = \frac{1}{\sqrt{1-\frac{1}{2}}} = \frac{1}{\frac{\sqrt{1}}{\sqrt{2}}} = \sqrt{2}$$

$$\boxed{y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)}$$

12. $y = \cos^{-1}(4x)$ at the point where $x = \frac{\sqrt{3}}{8}$

$$y_1 = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$m = -\frac{4}{\sqrt{1-16\left(\frac{\sqrt{3}}{8}\right)^2}} = -\frac{4}{\sqrt{\frac{1}{4}}} = -4\cdot\sqrt{4} = -8$$

$$\boxed{y - \frac{\pi}{6} = -8\left(x - \frac{\sqrt{3}}{8}\right)}$$

13. $y = \arctan(3x^2)$ at the point where $x = \frac{\sqrt{3}}{3}$

$y_1 = \tan^{-1}(3\cdot\frac{3}{9}) = \tan^{-1}(1) = \pi/4$

$m = \frac{6(\frac{\sqrt{3}}{3})}{(3(\frac{\sqrt{3}}{3})^2)^2+1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\boxed{y - \pi/4 = \sqrt{3}\left(x - \frac{\sqrt{3}}{3}\right)}$

14. $y = \sin^{-1}(5x)$ at the point where $x = -\frac{\sqrt{3}}{10}$

$y_1 = \sin^{-1}(-\frac{\sqrt{3}}{2}) = -\pi/3$

$m = \frac{5}{\sqrt{1-(-\frac{\sqrt{3}}{2})^2}} = \frac{5}{\sqrt{1-\frac{3}{4}}} = 5\cdot\frac{\sqrt{4}}{\sqrt{1}} = 10$

$\boxed{y + \pi/3 = 10\left(x + \frac{\sqrt{3}}{10}\right)}$

15. $y = \arccos(x)$ at the point where $x = -\frac{\sqrt{2}}{2}$

$y_1 = \cos^{-1}(-\frac{\sqrt{2}}{2}) = 3\pi/4$

$m = -\frac{1}{\sqrt{1-\frac{2}{4}}} = -\frac{1}{\frac{\sqrt{1}}{\sqrt{2}}} = -\sqrt{2}$

$\boxed{y - 3\pi/4 = -\sqrt{2}\left(x + \frac{\sqrt{2}}{2}\right)}$

16. $y = \arctan(x)$ at the point where $x = \sqrt{3}$

$y_1 = \tan^{-1}(\sqrt{3}) = \pi/3$

$m = \frac{1}{3+1} = \frac{1}{4}$

$\boxed{y - \pi/3 = \frac{1}{4}(x - \sqrt{3})}$

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Test Prep

17. Let $g(x) = (\arccos x^2)^5$. Then $g'(x) =$ $5(\cos^{-1}(x^2))^4 \cdot \left(-\frac{1}{\sqrt{1-x^4}}\right)\cdot(2x)$

(A) $-10\frac{(\arccos x^2)^4}{\sqrt{1-x^2}}$ (B) $-10\frac{x(\arccos x^2)^4}{\sqrt{1-x^4}}$ (C) $-10\frac{x(\arcsin x^2)^4}{\sqrt{1-x^2}}$

(D) $10\frac{x(\arccos x^2)^4}{\sqrt{1-x^2}}$ (E) $10\frac{(\arccos x^2)^4}{\sqrt{1-x^4}}$

18. If $\lim_{h\to 0}\frac{\arccos(a+h)-\arccos(a)}{h} = -3$, which of the following could be the value of a?

$f(x) = \cos^{-1}x \to f'(a) = -3 \to -\frac{1}{\sqrt{1-a^2}} = -3 \to \frac{1}{1-a^2} = 9$

$1 = 9 - 9a^2$

Solve!

(A) $\frac{\sqrt{8}}{3}$ (B) $\frac{1}{3}$ (C) 3 (D) 1

19. If $\arctan y = \ln x$, then $\frac{dy}{dx} =$ $\frac{1}{y^2+1}\cdot\frac{dy}{dx} = \frac{1}{x}$

(A) $\tan\left(\frac{1}{x}\right)$ (B) $\tan(\ln x)$ (C) $\frac{1+y^2}{xy}$ (D) $\frac{x}{1+y^2}$ (E) $\frac{1+y^2}{x}$