4.3 Rates of Change Other Than Motion

Calculus

Solutions **Practice**

1. A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function E defined by $E(t) = 0.3t^4 - 14t^3 + 110t^2$ for $0 \le t \le 12$. At what rate are shoppers entering the store 5 hours after the start of the sale?

$E'(5) = 200$ shoppers per hour

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2. The function $t = f(P)$ models the time, in days, for a small pond to evaporate as a function of the size P of the pond, measured in liters. What are the units for $f''(P)$?

days/ liter^2

3.

t (days)	$W(t)$ (°F)
0	58
5	60
10	61
15	64
20	68

The temperature, in degrees Fahrenheit (°F), of a lake is a differentiable function W of time t. The table above shows the water temperature as recorded every 5 days over a 20-day period. Use the data from the table to find an approximation for $W'(10)$. Show the computations that lead to your answer. Indicate units of measure.

$\frac{W(15) - W(5)}{15 - 5} = \frac{4}{10} = 0.4$ °F per day

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4. The function $D(t) = 13 - 2.7\cos\left(\frac{\pi}{4}t\right)$ models the depth, in feet, of water t hours after 6 A.M. Find the instantaneous rate of change of the depth of the water at 9 A.M. Use appropriate units.

$D'(3) \approx 1.499$ ft/hour

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5. The rate of consumption of liquid chocolate in Switzerland is given by $R(t) = ce^{kt}$, where R is measured in millions of gallons per year and t is measured in years from the beginning of 1990. The consumption rate doubles every 9 years and the consumption rate at the beginning of 1990 was 2 million gallons per year. Find c and k.

$2 = ce^{0.077(0)}$

$\boxed{2 = c}$

Double $\rightarrow$ $2c = ce^{k(9)}$

$2 = e^{9k}$

$\ln(2) = 9k$

$\boxed{k \approx 0.077}$

6.

t (minutes)	0	10	20	30
$W(t)$ (°F)	104	90	79	72

The temperature of water in a bathtub at time t is modeled by a strictly decreasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. The water is cooling for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t are given in the table above. Use the data in the table to estimate $W'(15)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(15) \approx \frac{W(20)-W(10)}{20-10} = \frac{-11}{10} = -1.1$$

The water in the bathtub is cooling at an estimated rate of -1.1 degrees Fahrenheit per minute at 15 minutes.

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7. The rate of decay, in grams per minute, of a radioactive substance is a differentiable, decreasing function R of time, t, in minutes. The table below shows the decay rate as recorded every 4 minutes over a 24-minute period.

t (minutes)	0	4	8	12	16	20	24
$R(t)$ (grams per minute)	320	221	130	82	39	22	11

a. Approximate $R'(12)$. Show the computations that lead to your answer. Indicate units of measure.

One of these!

$$\frac{W(16)-W(8)}{16-8} = \frac{39-130}{8} = \boxed{-11.375 \text{ grams}/\text{min}^2} \qquad \frac{W(16)-W(12)}{16-12} = \boxed{-10.75 \text{ g}/\text{min}^2}$$

$$\frac{W(12)-W(8)}{12-8} = \frac{82-130}{4} = \boxed{-12 \text{ grams}/\text{min}^2}$$

b. A physicist proposes the function $G(t) = 320(0.882)^t$ as a model for the rate of decay of the radioactive substance, using the same units as $R(t)$. Find $G'(12)$. Using appropriate units, explain the meaning of your answer in terms of the decay rate of the substance.

-8.905 grams/min/min. At time $t = 12$ minutes, the decay rate is decreasing at the rate of 8.905 grams per minute per minute.

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8. For time $t \geq 0$ hours, let $r(t) = 50\left(1 - e^{-0.7t^2}\right)$ represent the speed, in miles per hour, at which a car travels along a straight road. Find $r'(2)$. Indicate units of measure.

8.513 miles per hour^2

9. People are entering a museum at a rate modeled by $f(t)$ people per hour and exiting the building at a rate modeled by $g(t)$ people per hour, where t is measured in hours. The functions f and g are nonnegative and differentiable for all times t. Which of the following inequalities indicates that the rate of change of the number of people in the building is decreasing at time t?

When is the rate decreasing?

(A) $g(t) < 0$ (B) $g'(t) < 0$ (C) $f(t) - g(t) < 0$ (D) $f'(t) - g'(t) < 0$

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10. **Calculator Active.** The rate of $R(t)$ of a certain item produced in a factory is given by

$$R(t) = 4000 + 48(t - 3) - 4(t - 3)^3$$

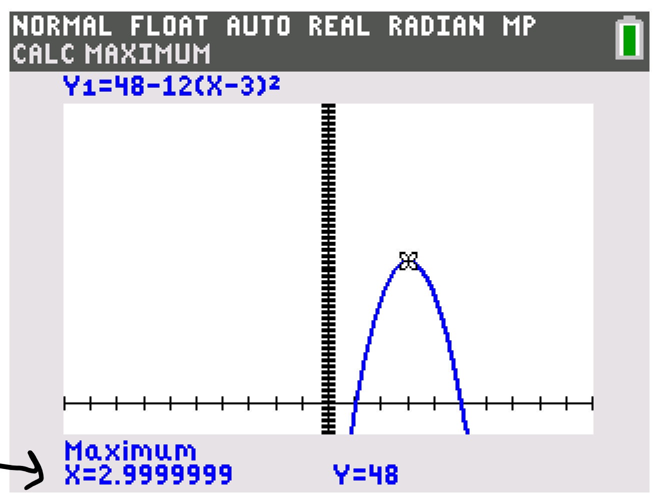
where t is the number of hours since the beginning of the workday at 8:00 a.m. At what time is the rate of production increasing most rapidly?

When is $R(t)$ increasing most? = $R'(t)$ has a max

$R'(t) = 48 - 12(t-3)^2$

(A) 8:00 am (B) 10:00 am (C) 11:00 am (D) 1:00 pm

Graph $R'(t)$ and find max.



$t = 3$

8 AM + 3 = 11 A.M