Calculus

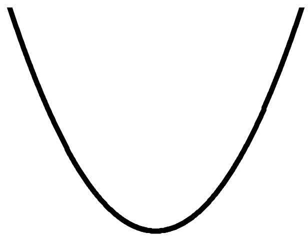
4.6 Approximating with Local Linearity

Notes

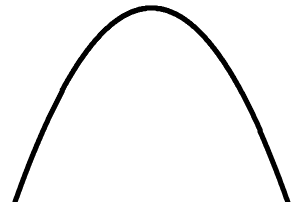
Write your questions and thoughts here!

The tangent line of the function $f(x)$ at $x = a$ can give you an approximate value of $f(x)$ for points close to $x = a$.

Concave UP with a Tangent Line



Concave DOWN with a Tangent Line



1. f is concave up on its domain with $f(4) = 5$ and $f'(4) = 3$.
   a. What is the estimate for $f(3.8)$ using the local linear approximation for f at $x = 4$?

   b. Is it an underestimate or overestimate? Explain.

2. The function $f(x) = 5x - 2x^3 - 2$ is concave down at $x = 1$?.
   a. Find the tangent line of f at $x = 1$.

   b. What is the estimate for $f(1.1)$ using the local linear approximation for f at $x = 1$?

   c. Is it an underestimate or overestimate? Explain.

3. Consider the differential equation $\frac{dy}{dx} = e^y(2x^2 - 5x)$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 0$.
   a. Write an equation for the line tangent to the graph of f at the point $(2,0)$.

   b. Use the tangent line to approximate $f(2.2)$.

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Practice

For each differential equation, let $y = f(x)$ be the particular solution to the differential equation with the given initial condition.

1. $\frac{dy}{dx} = (5 - y)\sin x$ and $f\left(\frac{\pi}{2}\right) = 2$.
   a. Write an equation for the line tangent to the graph of f at the point $\left(\frac{\pi}{2}, 2\right)$.

   b. Use the tangent line to approximate $f(1.5)$.

2. $\frac{dy}{dx} = -\frac{4x}{y}$ and $f(1) = 3$.
   a. Write an equation for the line tangent to the graph of f at the point $(1,3)$.

   b. Use the tangent line to approximate $f(1.1)$.

Answer the questions for each function listed.

3. $f(x) = 2\cos x + 1$ is concave down on $\left[0, \frac{\pi}{2}\right]$.
   a. What is the estimate for $f(1)$ using the local linear approximation for f at $x = \frac{\pi}{2}$? Give an exact answer (no rounding).

   b. Is it an underestimate or overestimate? Explain.

4. $f(x) = \frac{e^{2x}}{x+1}$ is concave up on $x > -1$.
   a. What is the estimate for $f(0.1)$ using the local linear approximation for f at $x = 0$?

   b. Is it an underestimate or overestimate? Explain.

5. $f(x) = -\sqrt{4-x}$ is concave up on its domain.
   a. What is the estimate for $f(1.9)$ using the local linear approximation for f at $x = 2$? Round to three decimal places.

   b. Is it an underestimate or overestimate? Explain.

6. f is concave down and $f(3) = -1$ and $f'(3) = 2$.
   a. What is the estimate for $f(3.2)$ using the local linear approximation for f at $x = 3$?

   b. Is it an underestimate or overestimate? Explain.

7. f is concave up and $f(-5) = 2$ and $f'(-5) = -1$.
   a. What is the estimate for $f(-5.1)$ using the local linear approximation for f at $x = -5$?

   b. Is it an underestimate or overestimate? Explain.

8. f is concave down and $f(2) = 1$ and $f'(2) = -3$.
   a. What is the estimate for $f(1.9)$ using the local linear approximation for f at $x = 2$?

   b. Is it an underestimate or overestimate? Explain.

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Test Prep

9. Let f be the function given by $f(x) = 3x^2 - 4x + 2$. The tangent line to the graph of f at $x = 1$ is used to approximate values of $f(x)$. Which of the following is the smallest value of x for which the error resulting from this tangent line approximation is more than 0.5?
[*Hint for your calculator use*: Create a table to compare values of two functions.]

(A) 1.3 (B) 1.4 (C) 1.5 (D) 1.6 (E) 1.7

10. The depth of snow in a field is given by the twice-differentiable function S for $0 \leq t \leq 12$, where $S(t)$ is measured in centimeters and time t is measured in hours. Values of $S'(t)$, the derivative of S, at selected values of time t are shown in the table above. It is known that the graph of S is concave down for $0 \leq t \leq 12$.

t (hours)	0	1	4	9	12
$S'(t)$ (centimeters per hour)	1.8	2.4	2.0	1.6	1.3

a. Use the data in the table to approximate $S''(10)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $S''(10)$ in the context of the problem.

b. Is there a time t, for $0 \leq t \leq 12$, at which the depth of snow is changing at a rate of 1.5 centimeters per hour? Justify your answer?

c. At time $t = 4$, the depth of snow is 28 centimeters. Use the line tangent to the graph of S at $t = 4$ to approximate the depth of the snow at time $t = 6$. Is the approximation an underestimate or an overestimate of the actual depth of snow at time $t = 6$? Justify your answer.