Calculus

5.11 Solving Optimization Problems

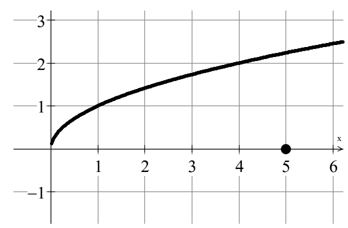
Notes

Write your questions and thoughts here!

Strategies for solving optimization problems:

1. Draw a picture (if applicable) and identify *known* and *unknown* quantities.
2. Write an equation (model) that will be optimized.
3. Write your equation in terms of a single variable.
4. Determine the desired max or min value with calculus techniques.
5. Determine the domain (endpoints) of your equation to verify if the endpoints represent a max or min.

1. What point on the graph $y = \sqrt{x}$ is closest to $(5,0)$.



2. Two towers are 30 feet apart. One is 12 feet high and the other is 28 feet high. There is a stake in the ground between the towers. The top of each tower has a wire tied to it that connects to the stake on the ground. Where should the stake be placed to use the least amount of wire?

3. A particle is traveling along the x-axis and its position from the origin can be modeled by
$$x(t) = t^3 - 15t^2 + 72t - 9$$
where x is centimeters and t is seconds.

   a. On the interval $3 \le t \le 9$, find when the particle is farthest to the right.

   b. On the same interval, what is the particle's maximum speed?

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Practice

1. A particle is traveling along the x-axis and it's position from the origin can be modeled by $x(t) = -\frac{2}{3}t^3 + t^2 + 12t + 1$ where x is meters and t is minutes on the interval .
   a. At what time t during the interval $0 \leq t \leq 4$ is the particle farthest to the left?

   b. On the same interval what is the particle's maximum speed?

2. Find the point on the graph of the function $f(x) = x^2$ that is closest to the point $\left(2, \frac{1}{2}\right)$.

3. A particle moves along the x-axis so that at any time t its position is $s(t) = \frac{1}{3}t^3 - 4t^2 + 7t - 5$ where s is inches and t is hours.
   a. At what time t during the interval $0 \leq t \leq 6$ is the particle farthest to the right?

   b. On the same interval what is the particle's maximum speed?

4. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 20 - x^2$. Find the dimensions of the rectangle with the largest area.

5. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material? Volume of a cylinder is $V = \pi r^2 h$. Surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$

6. A swimmer is 500 meters from the closest point on a straight shoreline. She needs to reach her house located 2000 meters down shore from the closest point. If she swims at $\frac{1}{2}$ m/s and she runs at 4 m/s, how far from her house should she come ashore so as to arrive at her house in the shortest time? *Hint*: $\text{time} = \frac{\text{distance}}{\text{rate}}$

7. Mr. Kelly is selling licorice for $1.50 per piece. The cost of producing each piece of licorice increases the more he produces. Mr. Kelly finds that the total cost to produce the licorice is $10\sqrt{x}$ dollars, where x is the number of licorice pieces. What is the most Mr. Kelly could lose per piece on the sale of licorice. Justify your answer. (hint: profit is the difference between money received and the cost of the licorice.)

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Test Prep

8. Let $f(x) = xe^{-x} + ce^{-x}$, where c is a positive constant. For what positive value of c does f have an absolute maximum at $x = -5$?

9. Let $f(x) = 9 - x^2$ for $x \geq 0$ and $f(x) \geq 0$. An isosceles triangle whose base is the interval from the point $(0, 0)$ to the point $(b, 0)$ has its vertex on the graph of f. For what value of b does the triangle have maximum area? Recall that the area of a triangle is modeled by $A = \frac{1}{2}(\text{base})(\text{height})$.

10. Mr. Sullivan is making apple juice from the apples he collected in his neighbor's orchard. The number of gallons of apple juice in a tank at time t is given by the twice-differentiable function A, where t is measured in days and $0 \leq t \leq 20$. Values of $A(t)$ at selected times t are given in the table below.

t (days)	0	3	8	12	20
$A(t)$ (gallons)	2	6	9	10	7

a. Use the data in the table to estimate the rate at which the number of gallons of apple juice in the tank is changing at time $t = 10$ days. Show the computations that lead to your answer. Indicate units of measure.

b. For $0 \leq t \leq 12$, is there a time t at which $A'(t) = \frac{2}{3}$? Justify your answer.

c. The number of gallons of apple juice in the tank at time t is also modeled by the function B defined by $B(t) = 3t - \frac{1}{2}(t+4)^{\frac{3}{2}} + 6$, where t is measured in days and $0 \leq t \leq 20$. Based on the model, at what time t, for $0 \leq t \leq 20$, is the number of gallons of apple juice in the tank an absolute maximum?

d. For the function B defined in part c, the locally linear approximation near $t = 5$ is used to approximate $B(5)$. Is this approximation an overestimate or an underestimate for the value of $B(5)$? Give a reason for your answer.