### 5.12 Behaviors of Implicit Relations

**Consider the curves in the xy-plane for each problem. At the point given point, is the curve increasing or decreasing? Justify your answer.**

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<td>1.</td>
<td>(x^2 - \frac{y^2}{2} = -1) at ((-1, 2))</td>
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\[
2x - \frac{y}{x} \frac{dy}{dx} = 0 \\
2(-1) - (2) \frac{dy}{dx} = 0 \\
-2 \frac{dy}{dx} = 2 \\
\frac{dy}{dx} = -1
\]
Decreasing because \(\frac{dy}{dx}(-1, 2) < 0\) |

| & 2. | \(x^2 + y^3 = 5\) at \((1, -8)\) | |  
\[
2x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0 \\
2 \frac{dy}{dx} + 3(-8)^2 \frac{dy}{dx} = 0 \\
12 \frac{dy}{dx} = -\frac{2}{3} \\
\frac{dy}{dx} = -\frac{1}{6}
\]
Increasing because \(\frac{dy}{dx}(1, -8) > 0\) |

| 3. | \(x^2 - 2xy + y^2 = 1\) at \((-1, -2)\) | |  
\[
2x - 2[(1) + x \frac{dy}{dx}] + 2y \frac{dy}{dx} = 0 \\
2(-1) - 2[-2 + (-1) \frac{dy}{dx}] + 2(-2) \frac{dy}{dx} = 0 \\
-2 + 4 + 2 \frac{dy}{dx} - 4 \frac{dy}{dx} = 0 \\
-2 \frac{dy}{dx} = -2 \\
\frac{dy}{dx} = 1
\]
Increasing because \(\frac{dy}{dx}(-1, -2) > 0\) |

| 4. | \(\frac{dy}{dx} = y \sin x\) where \(f(2\pi) = 1\) | |  
\[
\frac{d^2y}{dx^2} = \frac{dy}{dx} \sin x + y \cos x \\
(\sin x)(\sin x) + y \cos x \\
v \sin^2(x) + 1 \cos(x) \\
1 \cdot \sin^2(2\pi) + 1 \cdot \cos(2\pi) \\
0 + 1 \\
1
\]
Relative minimum b/c \(\frac{dy}{dx} = 0\) and \(\frac{d^2y}{dx^2} > 0\). |
5. \(\frac{dy}{dx} = \frac{x}{y} + \ln x\) where \(f(1) = -2\)

\[\frac{1}{2} + \ln(1)\]
\[-\frac{1}{2} + 0\]

Neither. \(\frac{dy}{dx} \neq 0\).

6. \(\frac{dy}{dx} = yx^2\) where \(f(0) = -5\)

\[(-5)(0) = 0\]
\[\frac{dy}{dx} = 0 \quad \checkmark\]

\[\frac{d^2y}{dx^2} = \frac{dy}{dx} x^2 + y (2x)\]
\[(yx^2)x^2 + 2xy\]
0 + 0

Neither. \(\frac{d^2y}{dx^2} = 0\)
therefore \(f\) is not concave up or down.

5.12 Behaviors of Implicit Relations

7. Consider the curve defined by \(x^2 - y^2 - 5xy = 25\).

a. Show that \(\frac{dy}{dx} = \frac{2x-5y}{5x+2y}\)

\[2x - 2y \frac{dy}{dx} - 5y - 5x \frac{dy}{dx} = 0\]
\[-2y \frac{dy}{dx} - 5x \frac{dy}{dx} = 5y - 2x\]
\[\frac{dy}{dx} (2y - 5x) = 5y - 2x\]

\[\frac{dy}{dx} = \frac{5y - 2x}{2y - 5x} = \frac{2x - 5y}{5x + 2y}\]

b. Find the slope of the line tangent to the curve at each point on the curve when \(x = 2\).

\[(2)^2 - y^2 - 5(2)y = 25\]
\[4 - y^2 - 10y = 25\]
\[-y^2 - 10y - 21 = 0\]
\[-(y^2 + 10y + 21) = 0\]
\[-(y + 3)(y + 7) = 0\]
\[y = -3 \quad y = -7\]

\[\frac{dy}{dx} (2, -3) = \frac{2(2) - 5(-3)}{5(2) + 2(-3)} = \frac{19}{4}\]

\[\frac{dy}{dx} (2, -7) = \frac{2(2) - 5(-7)}{5(2) + 2(-7)} = \frac{39}{4}\]
c. Find the positive value of $x$ at which the curve has a vertical tangent line. Show the work that leads to your answer.

**vertical tangent if** $\frac{dy}{dx}$ **has a denominator = 0,**

or $2y + 5x = 0.$

$2y = -5x$

$y = -\frac{5}{2}x$

Substitute “$y$” into original equation

$x^2 - (\frac{5}{2}x)^2 - 5x (\frac{5}{2}x) = 25$

$x^2 - \frac{25}{4} x^2 + \frac{25}{2} x^2 = 25$

$\frac{4}{4} x^2 - \frac{25}{4} x^2 + \frac{50}{4} x^2 = 25$

$\frac{25}{4} x^2 = 25$

$x^2 = \frac{100}{25}$

$x = \sqrt{\frac{100}{25}}$


d. Let $x$ and $y$ be functions of time $t$ that are related by the equation $x^2 - y^2 - 5xy = 25$. At time $t = 3$, the value of $x$ is 5, the value of $y$ is 0, and the value of $\frac{dy}{dt}$ is −2. Find the value of $\frac{dx}{dt}$ at time $t = 3$.

$$2x \frac{dx}{dt} - 2y \frac{dy}{dt} - 5 \left[ \frac{dx}{dt} y + x \frac{dy}{dt} \right] = 0$$

$$2(5) \frac{dx}{dt} - 2(0)(-2) - 5 \left[ \frac{dx}{dt} (0) + (5)(-2) \right] = 0$$

$$10 \frac{dx}{dt} - 5(-10) = 0$$

$$10 \frac{dx}{dt} = 50$$

$$\frac{dx}{dt} = -5$$