Calculus

5.1 The Mean Value Theorem

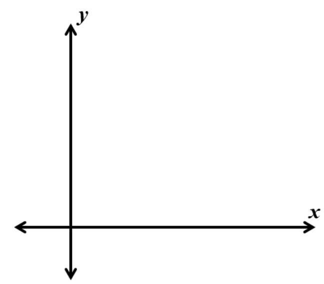
Notes

Write your questions and thoughts here!

We use the MVT to justify conclusions about a function over an interval.

Mean Value Theorem:

If a function f is continuous over the interval and differentiable over the interval then there exists a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.



1. Use the function $f(x) = -x^2 + 3x + 10$ to answer the following.
   a. On the interval $[2, 6]$, what is the average rate of change?

   b. On the interval $(2, 6)$, when does the instantaneous rate of change equal the average rate of change?

MVT vs IVT

Mean Value Theorem MVT	**Intermediate Value Theorem** IVT
• The derivative (instantaneous rate of change) must equal the average rate of change somewhere in the interval.	• On a given interval, you will have a y-value at each of the end points of the interval. Every y-value exists between these two y-values at least once in the interval.

2.

t minutes	0	5	15	20	30
$h(t)$ feet	0	40	70	65	80
$v(t)$ feet per minute	0	10	3	2	4

A hot air balloon is launched into the air with a human pilot. The twice-differentiable function h models the balloon's height, measured in feet, at time t, measured in minutes. The table above gives values of the $h(t)$ and the vertical velocity $v(t)$ of the balloon at selected times t.

a. For $5 \leq t \leq 20$, must there be a time t when the balloon is 50 feet in the air? Justify your answer.

b. For $20 \leq t \leq 30$, must there be a time t when the balloon's velocity is 1.5 feet per minute? Justify your answer.

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Practice

1. Skater Sully is riding a skateboard back and forth on a street that runs north/south. The twice-differentiable function S models Sully's position on the street, measured by how many meters north he is from his starting point, at time t, measured in seconds from the start of is ride. The table below gives values of the $S(t)$ and Sully's velocity $v(t)$ at selected times t.

t seconds	0	20	30	60
$S(t)$ meters	0	-5	7	40
$v(t)$ meters per second	0	3.2	0.8	-0.9

a. For $0 \leq t \leq 20$, must there be a time t when Sully is 2 meters south of his starting point? Justify your answer.

b. For $30 \leq t \leq 60$, must there be a time t when Sully's velocity is 1.1 meters per second? Justify your answer.

2. A particle is moving along the x-axis. The twice-differentiable function s models the particles distance from the origin, measured in centimeters, at time t, measured in seconds. The table below gives values of the $s(t)$ and the velocity $v(t)$ of the particle at selected times t.

t Seconds	3	10	20	25
$s(t)$ cm	5	-2	-10	8
$v(t)$ cm per second	-4	-2	3	-2

a. For $20 \leq t \leq 25$, must there be a time t when the particle is at the origin? Justify your answer.

b. For $3 \leq t \leq 10$, must there be a time t when the particle's velocity is -1.5 cm per second? Justify your answer.

3. A hot air balloon is launched into the air with a human pilot. The twice-differentiable function h models the balloon's height, measured in feet, at time t, measured in minutes. The table below gives values of the $h(t)$ and the vertical velocity $v(t)$ of the balloon at selected times t.

t minutes	0	6	10	40
$h(t)$ feet	0	46	35	105
$v(t)$ feet per minute	0	6	20	1

a. For $6 \leq t \leq 10$, must there be a time t when the balloon is 50 feet in the air? Justify your answer.

b. For $10 \leq t \leq 40$, must there be a time t when the balloon's velocity is 3 feet per second? Justify your answer.

Using the Mean Value Theorem, find where the instantaneous rate of change is equivalent to the average rate of change.

4. $y = x^2 - 5x + 2$ on $[-4, -2]$

5. $y = \sin 3x$ on $[0, \pi]$

6. $y = (-5x + 15)^{\frac{1}{2}}$ on $[1, 3]$

7. $y = e^x$ on $[0, \ln 2]$

5.1 The Mean Value Theorem

Test Prep

Calculator active problem

8. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = t^3 - 3t^2 + t + 1$. For what values of t, $0 \leq t \leq 2$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 2]$?

No calculator on this problem.

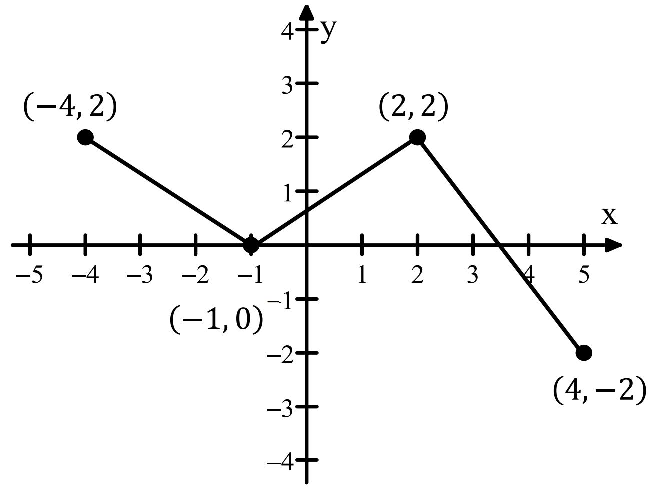
9. The table below gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$.

x	$f(x)$
3	12.5
5	13.9
7	16.1

Which of the following could be the value of $f'(5)$?

(A) 0.5 (B) 0.7 (C) 0.9 (D) 1.1 (E) 1.3

10. Let g be a continuous function. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-4 \le x \le 4$. Find the average rate of change of $g'(x)$ on the interval $-4 \le x \le 4$. Does the Mean Value Theorem applied on the interval $-4 \le x \le 4$ guarantee a value of c, for $-4 < x < 4$, such that $g''(c)$ is equal to this average rate of change? Why or why not?



Graph of g'

11.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	8	2	4
2	6	3	1	2
3	5	−3	6	3
4	−2	6	3	5

The functions f and g are differentiable for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f\big(g(x)\big) + 2$. Must there be a value c for $2 < c < 4$ such that $h'(c) = 1$.

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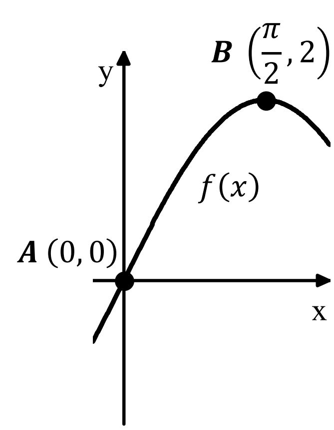
12. **Calculator active problem.** Let f be the function given by $f(x) = 2\sin x$. As shown above, the graph f crosses the origin at point A and point B at the coordinate point $\left(\frac{\pi}{2}, 2\right)$. Find the x-coordinate of the point on the graph of f, between points A and B, at which the line tangent to the graph of f is parallel to line AB. Round or truncate to three decimals.



13. A differentiable function g has the property that $g'(x) > 2$ for $1 \leq x \leq 5$ and $g(4) = 3$. Which of the following could be true?

I. $g(1) = -6$

II. $g(2) = 0$

III. $g(5) = 4$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) II and III only

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14. **Calculator active problem.** Let f be the function with $f(1) = e$, $f(4) = \frac{1}{e}$, and derivative given by $f'(x) = (x-1)\sin(ex)$. How many values of x in the open interval $(1, 4)$ satisfy the conclusion of the Mean Value Theorem for the function f on the closed interval $[1, 4]$?

(A) None

(B) One

(C) Two

(D) More than two