5.3 Increasing and Decreasing Intervals

Calculus

Practice

The following graphs show the derivative of f, f'. Identify the intervals when f is increasing and decreasing. Include a justification statement.

1.

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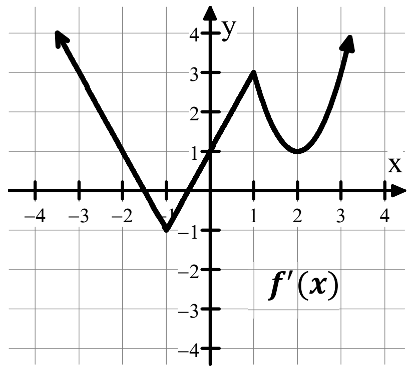
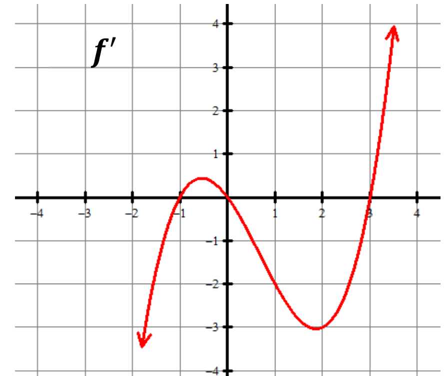


Increasing: $(-\infty, -1.5)$ and $(-0.5, \infty)$ because $f'(x) > 0$

Decreasing: $(-1.5, -0.5)$ b/c $f'(x) < 0$

2.



Increasing: $(-1, 0)$ and $(3, \infty)$ b/c $f'(x) > 0$.

Decreasing: $(-\infty, -1)$ and $(0, 3)$ b/c $f'(x) < 0$

For each function, find the intervals where it is increasing and decreasing, and JUSTIFY your conclusion. Construct a sign chart to help you organize the information, but do not use a calculator.

3. $f(x) = x^3 - 12x + 1$

$f'(x) = 3x^2 - 12 = 0$

$3x^2 = 12$

$x = \pm 2$

x	$(-\infty, -2)$	-2	$(-2, 2)$	2	$(2, \infty)$
$f'(x)$	pos	0	neg	0	pos

Decreasing on $(-2, 2)$ because $f'(x) < 0$

Increasing on $(-\infty, -2)$ and $(2, \infty)$ because $f'(x) > 0$

4. $g(x) = x^2(x - 3)$

$g'(x) = 2x(x-3) + x^2$

$2x^2 - 6x + x^2 = 0$

$x(3x - 6) = 0$

$x = 0 \quad x = 2$

x	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, \infty)$
$f'(x)$	pos	0	neg	0	pos

Increasing on $(-\infty, 0)$ and $(2, \infty)$ because $f'(x) > 0$.

Decreasing on $(0, 2)$ because $f'(x) < 0$.

5. $f(x) = x^2 e^x$

$f'(x) = 2xe^x + x^2 e^x$

$xe^x(2 + x) = 0$

$x = 0 \quad e^x = 0 \quad x + 2 = 0$

not possible $\quad x = -2$

x	$(-\infty, -2)$	-2	$(-2, 0)$	0	$(0, \infty)$
$f'(x)$	pos	0	neg	0	pos

Inc on $(-\infty, -2)$ and $(0, \infty)$ b/c $f'(x) > 0$

Dec on $(-2, 0)$ b/c $f'(x) < 0$

6. $g(t) = 12(1 + \cos t)$ on the interval $(0, 2\pi)$

$g'(t) = -12 \sin t$

$\sin t = 0$

$t = 0$

$t = \pi$

$t = 2\pi$

x	0	$(0, \pi)$	π	$(\pi, 2\pi)$	2π
$f'(x)$	0	neg	0	pos	0

Increasing on $(\pi, 2\pi)$ b/c $f' > 0$.

Dec. on $(0, \pi)$ b/c $f' < 0$.

The derivative f' is given for each problem. Use a calculator to help you answer each question about f.		
7. $f'(x) = \frac{x+3e^{-x}}{x^2+0.8}$. On what intervals is f increasing? $(-\infty, \infty)$	8. $f'(x) = -\sin x - x\cos x$ for $0 \le x \le \pi$. On which interval(s) is f decreasing? $(0, 2.0287)$	9. $f'(x) = \frac{1}{x} - e^x \sin x$ for $0 < x \le 4$. On what intervals is f decreasing? $(0.727, 3.1275)$

For #10-12, calculator use is encouraged.

10. The rate of money brought in by a particular mutual fund is represented by $m(t) = \left(\frac{e}{2}\right)^t$ thousand dollars per year where t is measured in years. Is the amount of money from this mutual fund increasing or decreasing at time $t = 5$ years? Justify your answer.

$m(5) \approx 4.6379$

Increasing because $m(5) > 0$.

11. The number of hair follicles on Mr. Sullivan's scalp is measured by the function $h(t) = 500e^{-t}$ where t is measured in years. Is the amount of hair increasing or decreasing at $t = 7$ years? Justify your answer.

$h'(7) = -0.4559$

Decreasing because $h'(7) < 0$.

12. The rate at which rainwater flows into a street gutter is modeled by the function $G(t) = 10\sin\left(\frac{t^2}{30}\right)$ cubic feet per hour where t is measured in hours and $0 \le t \le 8$. The gutter's drainage system allows water to flow out of the gutter at a rate modeled by $D(t) = -0.02x^3 + 0.05x^2 + 0.87x$ for $0 \le t \le 8$. Is the amount of water in the gutter increasing or decreasing at time $t = 4$ hours? Give a reason for your answer.

$G(4) - D(4) \approx 2.084$

Increasing b/c $G(4) - D(4) > 0$.

5.3 Increasing and Decreasing Intervals

Test Prep

13.

x	1	2	3	4	5
$f(x)$	-6	-1	3	6	8

The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

(A) f' is negative and decreasing for $1 \le x \le 5$.

(B) f' is negative and increasing for $1 \le x \le 5$.

(C) f' is positive and decreasing for $1 \le x \le 5$.

(D) f' is positive and increasing for $1 \le x \le 5$.

14. Let f be the function given by $f(x) = 4 - x$. g is a function with derivative given by

$$g'(x) = f(x)f'(x)(x-2) = 0$$

On what intervals is g decreasing?

x		2		4	
$g'(x)$	pos	0	neg	0	pos

$f(x) = 0$
$x = 4$

$f'(x) = 0$
$-1 \neq 0$

$x - 2 = 0$
$x = 2$

(A) $(-\infty, 2]$ and $[2, \infty)$ (B) $(-\infty, 2]$ only (C) $[2, 4]$ only

(D) $[2, \infty)$ only (E) $[4, \infty)$ only

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15. Particle X moves along the positive x-axis so that its position at time $t \geq 0$ is given by $x(t) = 2t^3 - 4t^2 + 4$.

(a) Is particle X moving toward the left or toward the right at time $t = 2$? Give a reason for your answer.

$x'(t) = 6t^2 - 8t$
$x'(2) = 24 - 16 = 8$

Right because $x'(t) > 0$.

(b) At what time $t \geq 0$ is particle X farthest to the left? Justify your answer.

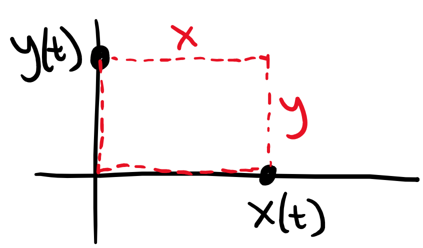
$6t^2 - 8t = 0$
$2t(3t-4) = 0$

$t = 0 \quad t = 4/3$

t	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$x'(t)$	neg.	0	pos.

At $t = 4/3$ because the particle moves right for $t > 4/3$

(c) A second particle, Y, moves along the positive y-axis so that its position at time t is given by $y(t) = 4t + 5$. At any time t, $t \geq 0$, the origin and the positions of the particles X and Y are the vertices of a rectangle in the first quadrant. Find the rate of change of the area of the rectangle at time $t = 2$. Show the work that leads to your answer.



$y(2) = 13$
$y'(2) = 4$

$A = x \cdot y$
$A' = x'y + xy'$
$A' = x'(2) \cdot y(2) + x(2) \cdot y'(2)$
$A' = (8)(13) + (4)(4)$

$\boxed{A'(2) = 120}$