Calculus

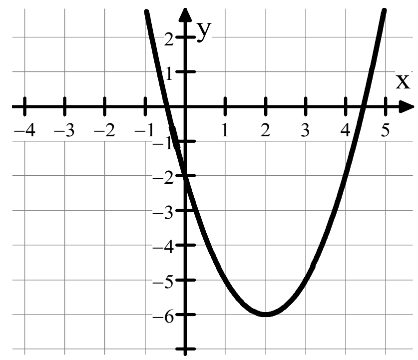
5.4 The First Derivative Test

Notes

Write your questions and thoughts here!

The First Derivative Test is when we use the first derivative to "test" whether or not a function has a maximum or minimum.

Start with something we know. A quadratic function's graph is a parabola. We know $f(x) = x^2 - 4x - 2$ opens up, so f will have a minimum. Examine the graph of this parabola and describe the behavior of $f'(x)$ around the minimum.



Justification statements

Assume c and d are critical numbers of a function f.

There is a **minimum** value at $x = c$ because

There is a **maximum** value at $x = d$ because

1. Use the First Derivative Test to find the x-values of any relative extrema of $f(x) = (x^2 - 4)^{\frac{2}{3}}$.

If $h(c)$ does not exist, then $x = c$ cannot be a critical point.

2. Find the relative max/min of the function $h(x) = \frac{x^2}{4-x}$

"What is the maximum value" is not the same as "where is the maximum".

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Calculus

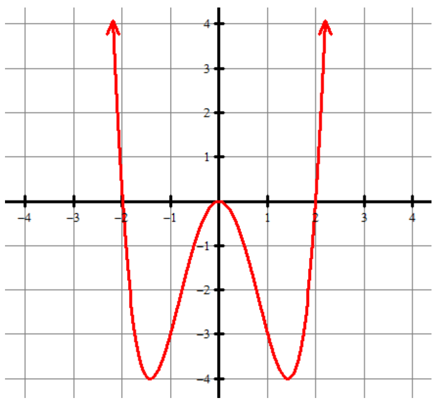
Practice

1. Assume $f(x)$ is continuous for all real numbers. The sign of its derivative is given in the table below for the domain of f. Identify all relative extrema and justify your answers.

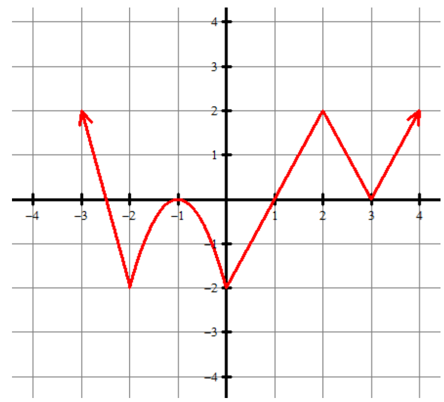
Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 3)$	$(3, \infty)$
$f'(x)$	**Positive**	**Negative**	**Negative**	**Positive**

For each problem, the graph of f', the derivative of f, is shown. Find all relative max/min of f and justify.

2.



3.



For each problem, the derivative of a function g is given. Find all relative max/min of g and justify.

4. $g'(x) = (x + 4)e^x$

5. $g'(x) = x^2 + 5x + 4$

Use a calculator to help find all x-values of relative max/min of f. No justification necessary.

6. $f'(x) = x^3 - 6\cos(x^2) + 2$

7. $f'(x) = \frac{2 - \ln x}{x^2}$

8. $f'(x) = \sqrt{x^4 + 2} + x^2 - 5x$

Use the First Derivative Test to locate the x-value of all extrema. Classify if it is a relative max or min and justify your answer.

9. $f(x) = x^3 - 12x + 1$

10. $g(x) = xe^{5x}$

11. $h(x) = \frac{x^3}{x+1}$

12. $f(x) = (x-5)^{\frac{2}{3}}$

13. What is the maximum value of $g(x) = 2\cos x$ on the open interval $(-\pi, \pi)$?

14. What is the relative minimum value of $h(x) = -x^3 + 6x^2 - 3$?

15. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - x - 12)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 4$.

(B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 4$.

(C) f has a relative maximum at $x = 3$ and a relative minimum at $x = -4$.

(D) f has a relative minimum at $x = 3$ and a relative maximum at $x = -4$.

(E) It cannot be determined if f has any relative extrema.

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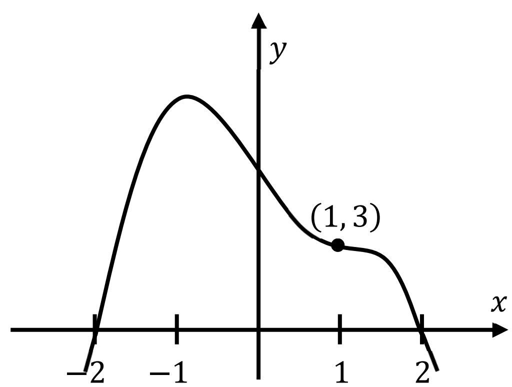
16. Let f be a twice-differentiable function defined on the interval $-2.1 < x < 2.1$ with $f(1) = -2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -2$ and $x = 2$ and has a horizontal tangent at $x = -1$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) Find the average rate of change of g', the derivative of g, over the interval $[-2,2]$.

(c) For $-2.1 < x < 2.1$, find all values of x at which g has a local minimum. Justify your answer.

(d) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative or zero? Justify your answer.