

## 6.10 Integrating with Long Division and Completing the Square

Calculus

Solutions

Practice

Find the indefinite integral.

1.  $\int \frac{4x^2}{x-2} dx$

$$\begin{array}{r} 2 \overline{) 4 \ 0 \ 0} \\ \underline{m \ 8 \ 16} \\ 4 \ 8 \ 16 \end{array}$$

$$\int 4x + 8 + \frac{16}{x-2} dx$$

$$2x^2 + 8x + 16 \ln|x-2| + C$$

2.  $\int \frac{28x^2 + 33x - 35}{4x+7} dx$

$$\begin{array}{r} 7x - 4 - \frac{7}{4x+7} \\ 4x+7 \overline{) 28x^2 + 33x - 35} \\ \underline{-(28x^2 + 49x)} \\ -16x - 35 \\ \underline{-(-16x - 28)} \\ -7 \end{array}$$

$$\int 7x - 4 - \frac{7}{4x+7} dx$$

$$u = 4x+7 \\ \frac{du}{4} = dx$$

$$\frac{7}{2}x^2 - 4x - \int \frac{7}{u} \frac{du}{4}$$

$$\frac{7}{2}x^2 - 4x - \frac{7}{4} \ln|4x+7| + C$$

3.  $\int \frac{1}{\sqrt{-x^2 - 4x - 3}} dx$

$$-(x^2 + 4x + 4) - 3 + 4$$

$$\int \frac{1}{\sqrt{1 - (x+2)^2}} dx$$

$$\sin^{-1}(x+2) + C$$

4.  $\int \frac{8}{x^2 - 10x + 26} dx$

$$(x^2 - 10x + 25) + 26 - 25$$

$$\int \frac{8}{(x-5)^2 + 1} dx$$

$$8 \tan^{-1}(x-5) + C$$

$$5. \int \frac{6}{\sqrt{-x^2-8x-7}} dx$$

$$-(x^2+8x+16)-7+16$$

$$\int \frac{6}{\sqrt{9-(x+4)^2}} dx \cdot \frac{\frac{1}{\sqrt{9}}}{\frac{1}{\sqrt{9}}}$$

$$\int \frac{2}{\sqrt{1-\left(\frac{x+4}{3}\right)^2}} dx \quad u = \frac{x+4}{3}$$

$$3du = dx$$

$$6 \sin^{-1}\left(\frac{x+4}{3}\right) + C$$

$$6. \int \frac{14x^2+11x-5}{2x+1} dx$$

$$2x+1 \overline{) \begin{array}{r} 7x+2-\frac{7}{2x+1} \\ 14x^2+11x-5 \\ \underline{-(14x^2+7x)} \end{array}}$$

$$\begin{array}{r} 4x-5 \\ \underline{-(4x+2)} \end{array}$$

$$-7$$

$$\int 7x+2-\frac{7}{2x+1}$$

$$u=2x+1$$

$$\frac{du}{2} = dx$$

$$\frac{7}{2}x^2 + 2x - \frac{7}{2} \ln|2x+1| + C$$

$$7. \int \frac{1}{x^2-12x+37} dx$$

$$(x^2-12x+36)+37-36$$

$$\int \frac{1}{(x-6)^2+1} dx$$

$$\tan^{-1}(x-6) + C$$

$$8. \int \frac{5x^3+12x^2-38x-54}{5x+7} dx$$

$$5x+7 \overline{) \begin{array}{r} x^2+x-9+\frac{9}{5x+7} \\ 5x^3+12x^2-38x-54 \\ \underline{-(5x^3+7x^2)} \end{array}}$$

$$\begin{array}{r} 5x^2-38x-54 \\ \underline{-(5x^2+7x)} \end{array}$$

$$-45x-54$$

$$\underline{-(-45x-63)}$$

$$9$$

$$\int x^2+x-9+\frac{9}{5x+7}$$

$$u=5x+7$$

$$\frac{du}{5} = dx$$

$$\frac{x^3}{3} + \frac{x^2}{2} - 9x + \frac{9}{5} \ln|5x+7| + C$$

$$9. \int \frac{3}{x^2+14x+49} dx$$

$$(x^2+14x+49)+49-49$$

$$\int \frac{3}{(x+7)^2} dx \quad \begin{array}{l} u=x+7 \\ du=dx \end{array}$$

$$\int 3u^{-2} du$$

$$\frac{3u^{-1}}{-1} + C = -\frac{3}{x+7} + C$$

$$10. \int \frac{x^3+2x^2+5}{x+2} dx$$

$$-2 \begin{array}{r|rrrr} 1 & 2 & 0 & 5 \\ x & -2 & 0 & 0 \\ \hline 1 & 0 & 0 & 5 \end{array}$$

$$\int x^2 + \frac{5}{x+2} dx$$

$$\frac{x^3}{3} + 5 \ln|x+2| + C$$

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### Test Prep

$$11. \int \frac{16}{x^2-6x+25} dx = \int \frac{16}{(x-3)^2+16} dx \cdot \frac{16}{16} = \int \frac{1}{\frac{(x-3)^2}{16}+1} dx$$

$$\int \frac{1}{\left(\frac{x-3}{4}\right)^2+1} dx \quad \begin{array}{l} u = \frac{1}{4}x - \frac{3}{4} \\ 4du = dx \end{array}$$

(A)  $16 \ln|x^2 - 16x + 25| + C$

(B)  $\tan^{-1}\left(\frac{x-3}{4}\right) + C$

(C)  $16 \tan^{-1}(x-3) + C$

(D)  $4 \tan^{-1}\left(\frac{x-3}{4}\right) + C$

$$12. \int \frac{12}{\sqrt{-x^2-2x+3}} dx = \int \frac{12}{\sqrt{4-(x+1)^2}} dx \cdot \frac{\frac{\sqrt{4}}{4}}{\frac{\sqrt{4}}{4}} = \int \frac{6}{\sqrt{1-\frac{(x+1)^2}{4}}} dx$$

$$\int \frac{6}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}} dx \quad \begin{array}{l} u = \frac{1}{2}x + \frac{1}{2} \\ 2du = dx \end{array}$$

(A)  $12 \sin^{-1}\left(\frac{x+1}{2}\right) + C$

(B)  $12 \sin^{-1}\left(\frac{x-1}{2}\right) + C$

(C)  $6 \sin^{-1}\left(\frac{x-1}{2}\right) + C$

(D)  $6 \sin^{-1}\left(\frac{x+1}{2}\right) + C$