6.11 Integration by Parts

Calculus

Solutions

Practice

Integrate the following.

1. $\int x\cos(x)dx$

$f(x)=x \quad g'(x)=\cos x$

$f'(x)=1 \quad g(x)=\sin x$

$x\sin x-\int(1)\sin x\,dx$

$x\sin x - -\cos x + C$

$x\sin x+\cos x+C$

2. $\int 2x\cos(3x+1)dx$

$f(x)=2x \quad g'(x)=\cos(3x+1)$

$f'(x)=2 \quad g(x)=\frac{1}{3}\sin(3x+1)$

$\frac{2}{3}x\sin(3x+1)-\int 2\left[\frac{1}{3}\sin(3x+1)\right]dx$

$\frac{2}{3}x\sin(3x+1)-\left(\frac{2}{3}\right)\left(-\frac{1}{3}\cos(3x+1)\right)+C$

$\frac{2}{3}x\sin(3x+1)+\frac{2}{9}\cos(3x+1)+C$

3. $\int x^2\sin(x)dx$

f		g'
x^2	$+$	$\sin x$
$2x$	$-$	$-\cos x$
2	$+$	$-\sin x$
0		$\cos x$

$-x^2\cos x+2x\sin x+2\cos x+C$

4. $\int 4xe^{3x+1}dx$

$f=4x \quad g'=e^{3x+1}$

$f'=4 \quad g=\frac{1}{3}e^{3x+1}$

$4x\left(\frac{1}{3}e^{3x+1}\right)-\int 4\left(\frac{1}{3}e^{3x+1}\right)dx$

$\frac{4}{3}xe^{3x+1}-\frac{4}{3}\cdot\frac{1}{3}e^{3x+1}+C$

$\frac{4}{3}xe^{3x+1}-\frac{4}{9}e^{3x+1}+C$

5. $\int_1^{e^2} x^4\ln x\,dx$

$f=\ln x \quad g'=x^4$

$f'=\frac{1}{x} \quad g=\frac{1}{5}x^5$

$\ln x\left(\frac{x^5}{5}\right)-\int_1^{e^2}\left(\frac{1}{x}\right)\left(\frac{x^5}{5}\right)dx$

$\frac{x^5}{5}\ln x\Big|_1^{e^2}-\frac{1}{5}\int_1^{e^2}x^4\,dx$

$\frac{x^5}{5}\ln x-\frac{x^5}{25}\Big|_1^{e^2}$

$\left(\frac{e^{10}}{5}\ln e^2-\frac{e^{10}}{25}\right)-\left(\frac{1}{5}\ln(1)-\frac{1}{25}\right)$

$\frac{2}{5}e^{10}-\frac{1}{25}e^{10}+\frac{1}{25}$

6. $\int \ln x\,dx$

$f=\ln x \quad g'=1$

$f'=\frac{1}{x} \quad g=x$

$x\ln x-\int\frac{1}{x}(x)dx$

$x\ln x-\int dx$

$x\ln x-x+C$

7. $\int_1^2 (3x^2 - 2x + 1)\ln x\, dx$

$f = \ln x \qquad g' = 3x^2 - 2x + 1$

$f' = \frac{1}{x} \qquad g = x^3 - x^2 + x$

$\ln x\,(x^3 - x^2 + x)\Big|_1^2 - \int_1^2 \left(\frac{1}{x}\right)(x^3 - x^2 + x)\,dx$

$\ln 2(8 - 4 + 2) - 0 - \int_1^2 (x^2 - x + 1)\,dx$

$6\ln 2 - \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]\Big|_1^2$

$6\ln 2 - \left[\left(\frac{8}{3} - 2 + 2\right) - \left(\frac{1}{3} - \frac{1}{2} + 1\right)\right]$

$6\ln 2 - \left[\frac{7}{3} - \frac{1}{2}\right]$

$6\ln 2 - \frac{11}{6}$

8. $\int x^3 e^x\, dx$

f		g'
x^3	+	e^x
$3x^2$	−	e^x
$6x$	+	e^x
6	−	e^x
0		e^x

$x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

9. The table gives values of f, f', g, and g' for selected values of x. If $\int_0^3 f'(x)g(x)\,dx = 6$, then $\int_0^3 f(x)g'(x)\,dx = ?$

x	0	3
$f(x)$	1	5
$f'(x)$	5	−3
$g(x)$	−4	3
$g'(x)$	3	2

$f \cdot g\Big|_0^3 - \int_0^3 f'g$

$f(3)g(3) - f(0)g(0) - 6$

$(5)(3) - (1)(-4) - 6$

$15 + 4 - 6 = 13$

10. Let f be a twice-differentiable function with selected values of f and its derivatives shown in the table. What is the value of $\int_0^3 xf''(x)\,dx$?

x	+	f''
1	−	f'
0		f

x	$f(x)$	$f'(x)$	$f''(x)$
0	2	−2	5
3	5	7	−2

$[xf'(x) - f(x)]\Big|_0^3$

$(3f'(3) - f(3)) - (0f'(0) - f(0))$

$(3(7) - 5) - (0 - 2) = (21 - 5) + 2 = 18$

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Test Prep

11. $\int x \cos 2x \, dx$

$f = x \qquad g' = \cos(2x)$

$f' = 1 \qquad g = \frac{1}{2}\sin(2x)$

$\frac{1}{2}x \sin(2x) - \int \frac{1}{2}\sin(2x)$

(A) $\frac{1}{2}x^2 \sin(2x) + C$

(B) $\frac{1}{2}x^2 \cos(2x) + \frac{1}{2}\sin(2x) + C$

(C) $\frac{1}{2}x \sin(2x) - \frac{1}{4}\cos(2x) + C$

(D) $\frac{1}{2}x \sin(2x) + \frac{1}{4}\cos(2x) + C$

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12. $\int_1^e x^4 \ln x \; dx$

$f = \ln x \qquad g = x^4$

$f' = \frac{1}{x} \qquad g = \frac{x^5}{5}$

$\frac{x^5}{5}\ln x \Big|_1^e - \int_1^e \frac{x^4}{5} dx$

$\left(\frac{e^5}{5}\ln e - 0\right) - \frac{x^5}{25}\Big|_1^e$

$\frac{e^5}{5} - \left[\frac{e^5}{25} - \frac{1}{25}\right]$

A) $\frac{6e^5-1}{25}$ (B) $\frac{4e^5+1}{25}$ (C) $\frac{1-e^3}{3}$ (D) e^4

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13. Let f be a differentiable function such that $\int f(x)\cos x \, dx = f(x)\sin x - \int \frac{1}{2}x^3 \sin x \, dx$. Which of the following could be $f(x)$.

$f \quad \cos x$

$f' \quad \sin x$

$f(x)\sin x - \int f'(x)\sin x$

$f'(x) = \frac{1}{2}x^3$

$f(x) = \frac{1}{2}\frac{x^4}{4} + C$

A) $\frac{1}{2}\sin x$ (B) $\frac{1}{2}\cos x$ (C) $\frac{1}{8}x^4$ (D) $\frac{1}{2}x^3$