

## 6.11 Integration by Parts

Calculus

## Solutions

## Practice

Integrate the following.

1.  $\int x \cos(x) dx$

$$\begin{aligned} f(x) &= x & g'(x) &= \cos x \\ f'(x) &= 1 & g(x) &= \sin x \end{aligned}$$

$$x \sin x - \int (1) \sin x dx$$

$$x \sin x - -\cos x + C$$

$$x \sin x + \cos x + C$$

2.  $\int 2x \cos(3x + 1) dx$

$$\begin{aligned} f(x) &= 2x & g'(x) &= \cos(3x+1) \\ f'(x) &= 2 & g(x) &= \frac{1}{3} \sin(3x+1) \end{aligned}$$

$$\frac{2}{3} x \sin(3x+1) - \int 2 \left[ \frac{1}{3} \sin(3x+1) \right] dx$$

$$\frac{2}{3} x \sin(3x+1) - \left(\frac{2}{3}\right) \left(-\frac{1}{3} \cos(3x+1)\right) + C$$

$$\frac{2}{3} x \sin(3x+1) + \frac{2}{9} \cos(3x+1) + C$$

3.  $\int x^2 \sin(x) dx$

$\frac{f}{x^2}$	$\frac{g'}{\sin x}$
2x	-cos x
2	-sin x
0	cos x

$$-x^2 \cos x + 2x \sin x + 2 \cos x + C$$

4.  $\int 4x e^{3x+1} dx$

$$\begin{aligned} f &= 4x & g' &= e^{3x+1} \\ f' &= 4 & g &= \frac{1}{3} e^{3x+1} \end{aligned}$$

$$4x \left(\frac{1}{3} e^{3x+1}\right) - \int 4 \left(\frac{1}{3} e^{3x+1}\right) dx$$

$$\frac{4}{3} x e^{3x+1} - \frac{4}{3} \cdot \frac{1}{3} e^{3x+1} + C$$

$$\frac{4}{3} x e^{3x+1} - \frac{4}{9} e^{3x+1} + C$$

5.  $\int_1^{e^2} x^4 \ln x dx$

$$\begin{aligned} f &= \ln x & g' &= x^4 \\ f' &= \frac{1}{x} & g &= \frac{1}{5} x^5 \end{aligned}$$

$$\ln x \left(\frac{x^5}{5}\right) - \int \left(\frac{1}{x}\right) \left(\frac{x^5}{5}\right) dx$$

$$\frac{x^5}{5} \ln x \Big|_1^{e^2} - \frac{1}{5} \int_1^{e^2} x^4 dx$$

$$\frac{1}{5} x^5 \ln x - \frac{x^5}{25} \Big|_1^{e^2}$$

$$\left(\frac{e^{10}}{5} \ln e^2 - \frac{e^{10}}{25}\right) - \left(\frac{1}{5} \ln(1) - \frac{1}{25}\right)$$

$$\frac{2}{5} e^{10} - \frac{1}{25} e^{10} + \frac{1}{25}$$

6.  $\int \ln x dx$

$$\begin{aligned} f &= \ln x & g' &= 1 \\ f' &= \frac{1}{x} & g &= x \end{aligned}$$

$$x \ln x - \int \frac{1}{x} (x) dx$$

$$x \ln x - \int dx$$

$$x \ln x - x + C$$

7.  $\int_1^2 (3x^2 - 2x + 1) \ln x \, dx$

$f = \ln x \quad g' = 3x^2 - 2x + 1$

$f' = \frac{1}{x} \quad g = x^3 - x^2 + x$

$\ln x (x^3 - x^2 + x) \Big|_1^2 - \int_1^2 \left(\frac{1}{x}\right)(x^3 - x^2 + x) \, dx$

$\ln 2(8 - 4 + 2) - 0 - \int_1^2 (x^2 - x + 1) \, dx$

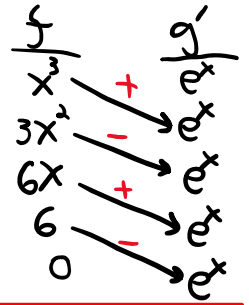
$6 \ln 2 - \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_1^2$

$6 \ln 2 - \left[\left(\frac{8}{3} - 2 + 2\right) - \left(\frac{1}{3} - \frac{1}{2} + 1\right)\right]$

$6 \ln 2 - \left[\frac{7}{3} - \frac{1}{2}\right]$

$6 \ln 2 - \frac{11}{6}$

8.  $\int x^3 e^x \, dx$



$x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$

9. The table gives values of  $f$ ,  $f'$ ,  $g$ , and  $g'$  for selected values of  $x$ . If  $\int_0^3 f'(x)g(x) \, dx = 6$ , then  $\int_0^3 f(x)g'(x) \, dx = ?$

$x$	0	3
$f(x)$	1	5
$f'(x)$	5	-3
$g(x)$	-4	3
$g'(x)$	3	2

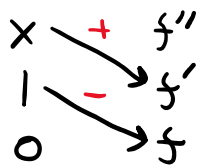
$f \cdot g \Big|_0^3 - \int_0^3 f' g$

$f(3)g(3) - f(0)g(0) - 6$

$(5)(3) - (1)(-4) - 6$

$15 + 4 - 6 = 13$

10. Let  $f$  be a twice-differentiable function with selected values of  $f$  and its derivatives shown in the table. What is the value of  $\int_0^3 x f''(x) \, dx$ ?



$x$	$f(x)$	$f'(x)$	$f''(x)$
0	2	-2	5
3	5	7	-2

$[x f'(x) - f(x)] \Big|_0^3$

$(3 f'(3) - f(3)) - (0 f'(0) - f(0))$

$(3(7) - 5) - (0 - 2) = (21 - 5) + 2 = 18$

**6.11 Integration by Parts**

11.  $\int x \cos 2x \, dx$

$f = x$        $g' = \cos(2x)$   
 $f' = 1$        $g = \frac{1}{2} \sin(2x)$

- (A)  $\frac{1}{2}x^2 \sin(2x) + C$
- (B)  $\frac{1}{2}x^2 \cos(2x) + \frac{1}{2} \sin(2x) + C$
- (C)  $\frac{1}{2}x \sin(2x) - \frac{1}{4} \cos(2x) + C$
- (D)  $\frac{1}{2}x \sin(2x) + \frac{1}{4} \cos(2x) + C$**

$\frac{1}{2}x \sin(2x) - \int \frac{1}{2} \sin(2x)$

12.  $\int_1^e x^4 \ln x \, dx$

$f = \ln x$        $g' = x^4$   
 $f' = \frac{1}{x}$        $g = \frac{x^5}{5}$

$\frac{x^5}{5} \ln x \Big|_1^e - \int_1^e \frac{x^4}{5} \, dx$   
 $\left( \frac{e^5}{5} \ln e - 0 \right) - \frac{x^5}{25} \Big|_1^e$   
 $\frac{e^5}{5} - \left[ \frac{e^5}{25} - \frac{1}{25} \right]$

- A)  $\frac{6e^5 - 1}{25}$
- (B)  $\frac{4e^5 + 1}{25}$**
- (C)  $\frac{1 - e^3}{3}$
- (D)  $e^4$

13. Let  $f$  be a differentiable function such that  $\int f(x) \cos x \, dx = f(x) \sin x - \int \frac{1}{2} x^3 \sin x \, dx$ . Which of the following could be  $f(x)$ .

$f = \cos x$   
 $f' = \sin x$

$f(x) \sin x - \int f'(x) \sin x$

$f'(x) = \frac{1}{2} x^3$   
 $f(x) = \frac{1}{2} \frac{x^4}{4} + C$

- A)  $\frac{1}{2} \sin x$
- (B)  $\frac{1}{2} \cos x$
- (C)  $\frac{1}{8} x^4$**
- (D)  $\frac{1}{2} x^3$