6.2 Approximating Areas with Riemann Sums

Calculus

Solutions

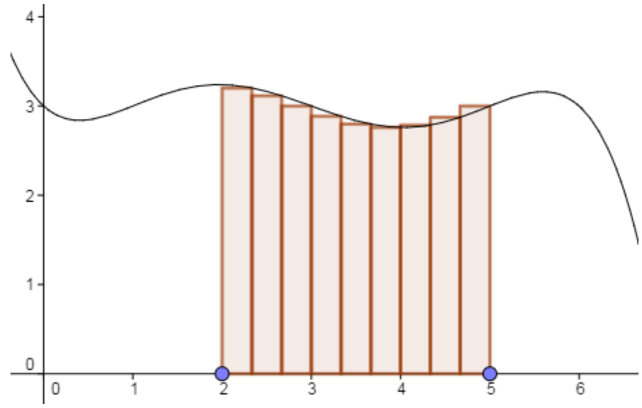
Practice

1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

Right

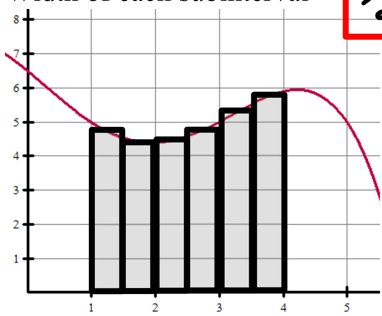
2. What is the width of each rectangle?

$\frac{5-2}{9} = \frac{3}{9} = \boxed{\frac{1}{3}}$

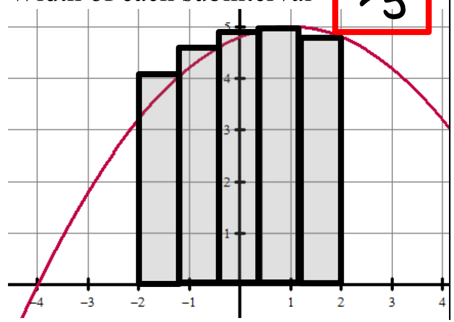


Sketch the following rectangular approximations. Find the width of each subinterval.

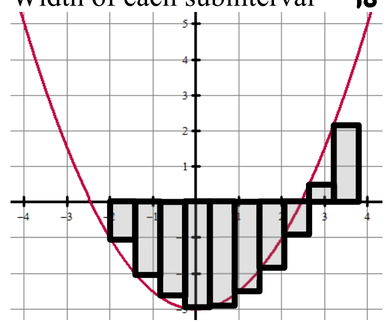
3. Midpoint on the interval [1,4] with $n = 6$ subintervals
Width of each subinterval = $\boxed{\frac{1}{2}}$



4. Right Endpoint on [-2,2] with $n = 5$ subintervals
Width of each subinterval = $\boxed{\frac{4}{5}}$



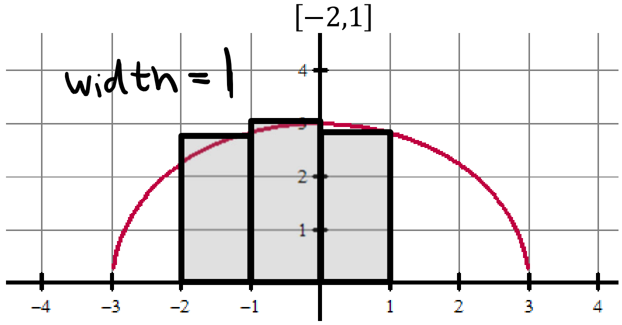
5. Left Endpoint on [-2,4] with $n = 10$ subintervals
Width of each subinterval = $\frac{6}{10} = \boxed{\frac{3}{5}}$



Approximate the area under the curve using the given Riemann Sum. Include a sketch!

6. $f(x) = \sqrt{9 - x^2}$
Right Riemann sum with 3 subintervals on the interval $[-2,1]$

width = 1

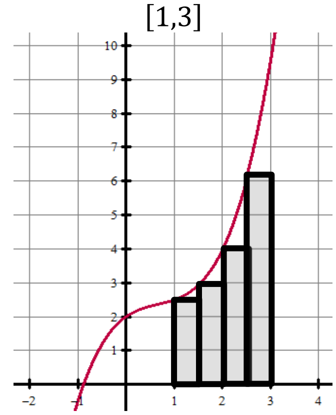


$1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1)$

$\sqrt{8} + 3 + \sqrt{8}$

$\approx \boxed{8.6568}$

7. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$
Left Riemann sum with 4 subintervals on the interval $[1,3]$



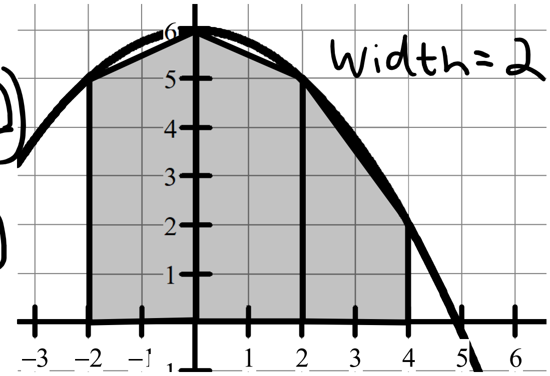
width = $\frac{1}{2}$

$\frac{1}{2} \cdot f(1) + \frac{1}{2} \cdot f(1.5) + \frac{1}{2} \cdot f(2) + \frac{1}{2} \cdot f(2.5)$

$\frac{1}{2}(2.5) + \frac{1}{2} \cdot (2.9375) + \frac{1}{2}(4) + \frac{1}{2}(6.0625)$

$\boxed{7.75}$

8. $f(x) = 6 - \frac{1}{4}x^2$ Trapezoidal approximation with 3 subintervals on the interval $[-2, 4]$.



$2 \cdot \left(\frac{f(-2)+f(0)}{2}\right) + 2 \cdot \left(\frac{f(0)+f(2)}{2}\right) + 2 \cdot \left(\frac{f(2)+f(4)}{2}\right)$

$f(-2) + f(0) + f(0) + f(2) + f(2) + f(4)$

$f(-2) + 2f(0) + 2f(2) + f(4)$

$5 + 2(6) + 2(5) + 2$

$\boxed{29}$

6.2 Approximating Areas with Riemann Sums

Test Prep

9. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable increasing function of t. The table shows the population change in people per year recorded at selected times.

Time years	0	4	10	13	20
$y(t)$ people per year	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

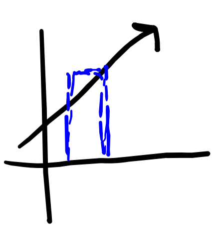
$4 \cdot (2724) + 6 \cdot (3108) + 3 \cdot (3697) + 7 \cdot (4283)$

70,616 people

b. What does your answer from part (a) represent?

The town has 70,616 more people after the 20-year period.

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value? Why?



Greater than the true value b/c it is a right Riemann sum on an increasing function.

10. A rectangular pool gets deeper from one end of the pool to the other. The table shows the depth $h(x)$ of the water at 4-foot intervals from one end of the pool to the other.

position, x feet	0	4	8	12	16	20	24	28	32
$h(x)$ feet	6.5	8	9.5	10	11	11.5	12	13	13.5

a. Use the data from the table to find an approximation for $h'(10)$, and explain the meaning of $h'(10)$ in terms of the depth of the pool. Show the computations that lead to your answer.

$$h'(10) \approx \frac{10-9.5}{12-8} = \frac{0.5}{4} = \boxed{0.125}$$

At 10 feet from one side of the pool, the depth is changing by 0.125 ft for every foot from one side of the pool.

b. Use a midpoint Riemann Sum with 4 subintervals to approximate the area under the curve from 0 to 32 feet.

$$8\cdot(8) + 8\cdot(10) + 8\cdot(11.5) + 8\cdot(13)$$

$$\boxed{340 \text{ feet}^2}$$

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11. The rate at which customers are being served at StarBrusts is given by the continuous function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 10$ hours, is given below. At $t = 0$ there had already been 200 customers served.

Time hours	0	2	3	6	10
$R(t)$ people/hour	37	44	36	42	48

Use a trapezoidal sum with four subintervals to approximate how many customers had been served after 10 hours.

$$2\cdot\left(\frac{37+44}{2}\right) + 1\cdot\left(\frac{44+36}{2}\right) + 3\cdot\left(\frac{36+42}{2}\right) + 4\cdot\left(\frac{42+48}{2}\right)$$

$$81 + 40 + 117 + 180$$

$$418$$

$$\boxed{618 \text{ customers}}$$