Calculus

6.5 Behavior of Accumulation Functions

Notes

Write your questions and thoughts here!

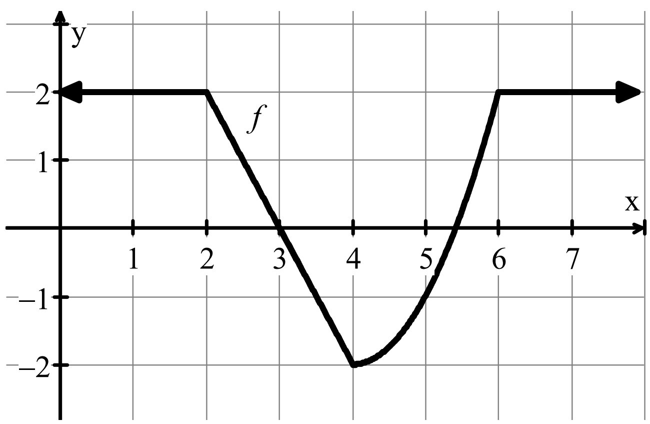
Accumulation Functions are of the form $F(x) = \int_a^x f(t)\,dt$, where a is a constant. Recognize that $F'(x) = f(x)$.

Today's lesson is a review of a lot of things from Unit 5. We will analyze the first and second derivative to understand the behavior of a function.

Behavior of Accumulation Functions

$F(x)$ is/has…	…when	or…
increasing		
decreasing		
relative max		
relative min		
concave up		
concave down		
a point of inflection		

1. Let $g(x) = \int_a^x f(t)\,dt$ where the graph of f is shown below and a is a constant.

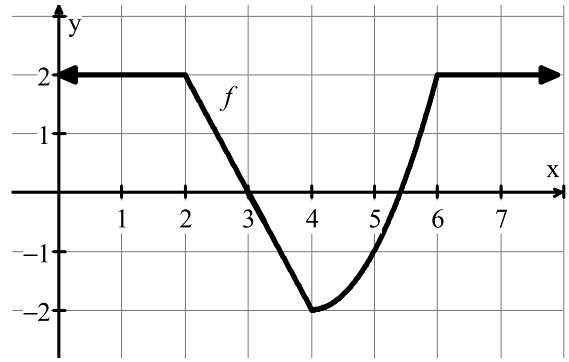


Find the following x-values.

a. Relative minimum of g.	b. Relative maximum of g.	c. Intervals where g is concave up.
d. Intervals where g is concave down.	e. Point of inflection(s) of g.	f. If $g(1) = 5$, what is the maximum value of g on the interval $[1, 5]$?

Write your questions and thoughts here!

2. Let $g(x) = \int_0^{\frac{x}{2}+5} f(t)\,dt$ where the graph of f is shown to the right. Find the x-value where g has a relative maximum.

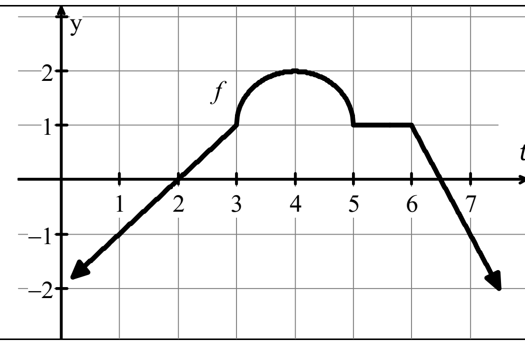


6.5 Behavior of Accumulation Functions

Calculus

Practice

1. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.

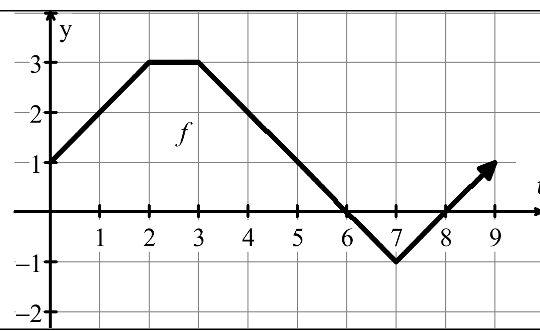


a. Relative minimum(s)	b. Relative maximum(s)
c. Concave up	d. Concave down

e. Increasing	f. Decreasing	g. Point(s) of inflection

h. If $g(3) = -2$, what is the maximum value of g on the interval $[3, 7]$?	i. Given $h(x) = \int_0^{2x-7} f(t)\,dt$. Find the x-value where h has a relative minimum.

2. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.



a. Relative minimum(s)	b. Relative maximum(s)
c. Concave up	d. Concave down

e. Increasing	f. Decreasing	g. Point(s) of inflection

h. If $g(2) = 1$, what is the maximum value of g on the interval $[2, 9]$?	i. Given $h(x) = \int_0^{\frac{x}{2}+5} f(t)\,dt$. Find the x-value where h has a relative minimum.

3. **Calculator active problem.** Let f be the function given by $f(x) = \int_{1/10}^{x} \sin\left(\frac{1}{t}\right) dt$ for $\frac{1}{10} < x < 1$. At what value(s) of x does f attain a relative maximum?

4. **Calculator active problem.** Let h be the function given by $h(x) = \int_{1}^{x} (1 - e^{\cos t})\, dt$ for $1 < x < 10$. At what value(s) of x does h attain a relative minimum?

5.

x	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$	4	$4 < x < 5$
$f(x)$	2	Positive	0	Negative	-3	Negative	0	Positive
$f'(x)$	-1	Negative	0	Negative	DNE	Positive	0	Negative
$f''(x)$	1	Positive	0	Negative	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[1, 5)$. The function f is twice differentiable except at $x = 3$. The function f and its derivatives have the properties indicated in the table above.

Let g be the function defined by $g(x) = \int_{2}^{x} f(t)\, dt$ on the open interval $(1, 5)$.

a. For $1 < x < 5$, find all critical points of g.

b. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

c. For $1 < x < 5$, find all values of x at which g has a point of inflection.

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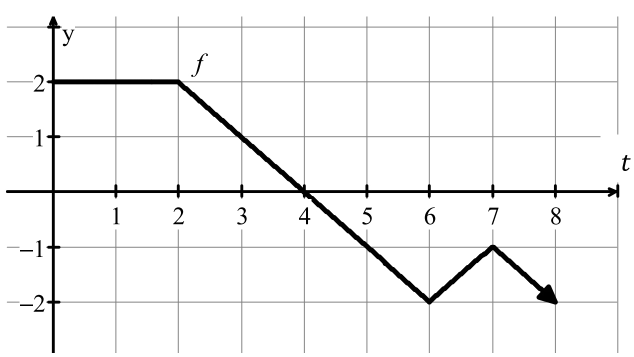
Test Prep

6.

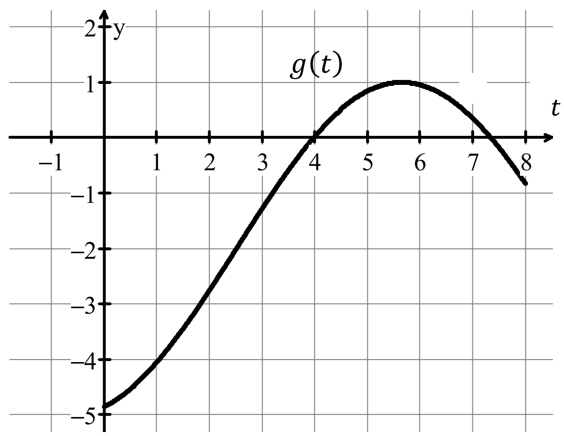


The graph of the function f is shown above. Let $g(x) = \int_{0}^{x} f(t)\, dt$.

a. Find the value of $g'(6)$.

b. Find the value of $g''(6)$.

7.



The graph of a differentiable function g is shown above. If $h(x) = \int_0^x g(t)\,dt$, which of the following is true?

(A) $h(4) < h'(4) < h''(4)$

(B) $h(4) < h''(4) < h'(4)$

(C) $h'(4) < h(4) < h''(4)$

(D) $h''(4) < h(4) < h'(4)$

(E) $h''(4) < h'(4) < h(4)$