Practice

1. Find an expression for the length of the curve $y = \sin x$ from $x = 0$ to $x = \frac{5\pi}{6}$. Do Not Evaluate.

$y' = \cos x$

$$\boxed{L = \int_0^{5\pi/6} \sqrt{1+\cos^2 x}\, dx}$$

2. The length of a curve from $x = 1$ to $x = 3$ is given by $\int_1^3 \sqrt{1 + 4x^2}\, dx$. If the point $(1, 6)$ is on the curve, which of the following could be an equation for this curve?

A. $y = \frac{4}{3}x^3 + x + 1$

B. $y = 4x^2 + 1$

C. $y = x^2 + 5$

D. $y = x^2 - 6$

E. $1 + \frac{4}{3}x^3$

$[y']^2 = 4x^2$

$y' = 2x$

$y = x^2 + C$

$1 + C$

$5 = C$

3. **Calculator active.** Suppose $G(x) = \int_0^x \sqrt{\sin(t)}\, dt$, for $0 \le x \le \pi$. What is the length of the arc along the curve $y = G(x)$ for $x = 0$ to $x = \pi/7$.

$G'(x) = \sqrt{\sin x}$

$L = \int_0^{\pi/7} \sqrt{1 + \sin x}\, dx \approx \boxed{0.495}$

4. **No Calculator.** Let $g(x) = \sqrt{3x}$ and f be an antiderivative of g.

a. Find $f'(x)$

$f'(x) = g(x) = \boxed{\sqrt{3x}}$

b. Find an expression for the length of the graph of f from $x = a$ to $x = b$.

$\boxed{\int_a^b \sqrt{1 + 3x}\, dx}$

c. If $a = 0$ and $b = 8$, find the length of the graph of f from a to b.

$u = 1 + 3x$

$\frac{du}{3} = dx$

$\int_1^{25} \sqrt{u}\, \frac{du}{3}$

$\frac{1}{3} u^{3/2} \cdot \frac{2}{3} \Big|_1^{25}$

$\frac{2}{9}\left[25^{3/2} - 1^{3/2}\right]$

$\frac{2}{9}\left[5^3 - 1\right]$

$\frac{2}{9}[124]$

$\boxed{\frac{248}{9}}$

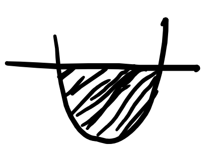
5. **Calculator active.** Consider the region bounded by the graphs of $f(x) = x^2 - 4$ and $g(x) = 5$.

a. Write an expression using one or more integrals that could be used to find the perimeter of this region.

$x^2 - 4 = 5$

$x^2 = 9$

$x = \pm 3$



$\boxed{\int_{-3}^{3} \sqrt{1 + (2x)^2}\, dx + \int_{-3}^{3} \sqrt{1 + (0)^2}\, dx}$

b. Find the perimeter.

$\boxed{25.494}$

6. Find an integral that gives the length of the graph $y = \cos\sqrt{x}$ between $x = a$ and $x = b$, where $0 < a < b$.

$y' = -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$$\boxed{\int_a^b \sqrt{1 + \frac{1}{4x}\sin^2\sqrt{x}}\, dx}$$

7. **Calculator active.** Let f be a function with derivative $f'(x) = \sqrt{x^5 + 1}$. What is the length of the graph of $y = f(x)$ from $x = 0$ to $x = 2.5$?

$$\int_0^{2.5} \sqrt{1 + (x^5 + 1)}\, dx \simeq \boxed{8.688}$$

8. Find an integral that is equal to the length of the curve $f(x) = \frac{5x^3 - 2x - 1}{7}$ from the point $(0, -0.143)$ to the point $(2, 5)$.

$f' = \frac{15}{7}x^2 - \frac{2}{7}$

$$\boxed{\int_0^2 \sqrt{1 + \left(\frac{15x^2 - 2}{7}\right)^2}\, dx} \quad \text{or} \quad \boxed{\frac{1}{7}\int_0^2 \sqrt{49 + (15x^2 - 2)^2}\, dx}$$

9. Find an expression for the length of the graph of $y = e^{3x}$ between $x = 1$ and $x = 3$.

$y' = 3e^{3x}$

$[3e^{3x}]^2 = 9e^{6x}$

$$\boxed{\int_1^3 \sqrt{1 + 9e^{6x}}\, dx}$$

10. **Calculator active.** The trajectory of a ball thrown from a height of 160 meters is given by the equation $y = 160 - \frac{x^2}{40}$ until it hits the water where y is the height of the ball above the water and x is the horizontal distance traveled in meters. Find the distance traveled by the ball from the time it is thrown until it hits the water.

$0 = 160 - \frac{x^2}{40}$

$x = 80$

$$\int_0^{80} \sqrt{1 + \left(-\frac{x}{20}\right)^2}\, dx \simeq \boxed{185.871}$$

8.13 Arc Length

Test Prep

11. Which of the following integrals gives the length of the curve $y = \frac{1}{2}x^3$ from $x = 1$ to $x = 3$?

$y' = \frac{3}{2}x^2$

$$\int_1^3 \sqrt{1 + \left(\frac{3}{2}x^2\right)^2}\, dx$$

$$\int_1^3 \sqrt{1 + \frac{9}{4}x^4}\, dx \rightarrow \int_1^3 \sqrt{\frac{1}{4}(4 + 9x^4)}\, dx$$

A. $\int_1^3 \sqrt{1 + \frac{1}{4}x^6}\, dx$

B. $\int_1^3 \sqrt{1 + \frac{1}{2}x^6}\, dx$

C. $\int_1^3 \frac{1}{2}\sqrt{4 + 9x^4}\, dx$

D. $\int_1^3 \sqrt{1 + \frac{3}{2}x^4}\, dx$

12. **Calculator active**. What is the length of the curve $y = 1 - \sin x$ from $x = 0$ to $x = 4\pi$?

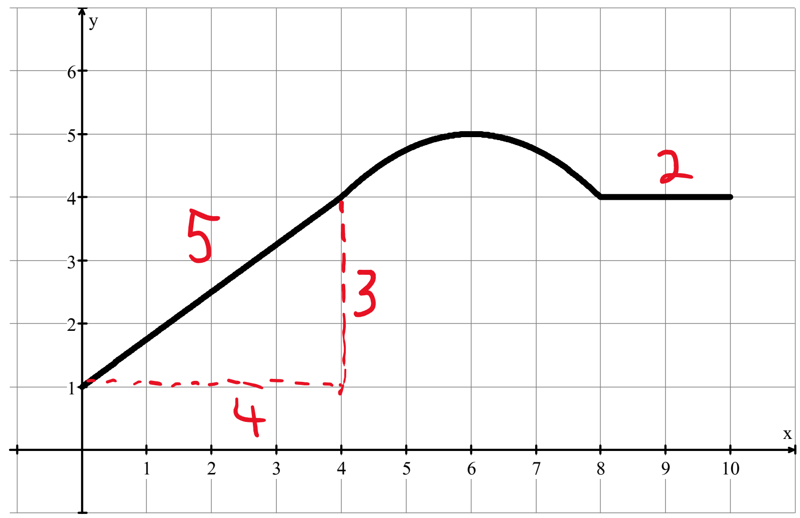
$y' = -\cos x$

$[-\cos x]^2 = \cos^2 x$

$\int_0^{4\pi} \sqrt{1 + \cos^2 x}\, dx$

$\boxed{15.2807}$

13.



$$f(x) = \begin{cases} 1 + \frac{3}{4}x & \text{for } 0 \le x < 4 \\ 5 - \frac{1}{4}(x-6)^2 & \text{for } 4 \le x < 8 \\ 4 & \text{for } 8 \le x \le 10 \end{cases}$$

A mountain hike consists of a steady incline followed by a curved hill and then a flat valley. The mountain hike is modeled by the piecewise-defined function f above, and the graph of f is shown in the figure above. Which of the following expressions gives the total length of the hike from $x = 0$ to $x = 10$.

$\frac{d}{dx}\left(5 - \frac{1}{4}(x-6)^2\right) = -\frac{1}{2}(x-6)$ $\quad [f']^2 = \frac{1}{4}(x-6)^2$

A. $2 + \int_0^8 \sqrt{1 + \left(\frac{3}{4} - \frac{1}{2}(x-6)\right)^2}\, dx$

B. $2 + \int_0^8 \sqrt{1 + \left(\frac{3}{4}\right)^2} + \sqrt{1 - \frac{1}{4}(x-6)^2}\, dx$

C. $7 + \int_4^8 \sqrt{1 + \left(1 - \frac{1}{2}(x-6)^2\right)^2}\, dx$

D. $7 + \int_4^8 \sqrt{1 + \frac{1}{4}(x-6)^2}\, dx$