

Write your questions
and thoughts here!

Recall:**Intermediate Value Theorem:****Average Rate of Change:**

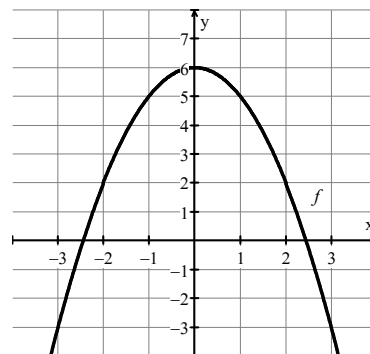
$$\frac{f(b) - f(a)}{b - a}$$

Mean Value Theorem (MVT):

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Average Value of a Function: The average value of a function on the interval $[a, b]$

1. Find the average value of $f(x) = 6 - x^2$ on $[-1, 3]$.



When does the function assume this value?

Comparing *average rate of change* and *average value* of a function.

Set up the equation for each question and use a calculator to solve it.

2. $h(t) = -16t^2 + 41t + 10$. h is height (feet) and t is time (seconds).
- What is the average height during the first 3 seconds?
 - What is the average velocity during the first 3 seconds?
3. $r(x) = 2 \sin x - 1$, where r is the rate at which Mr. Brust's waistline is changing (inches per month) and x is time (months).
- What is the average rate of change that Mr. Brust's waistline changes from the 10th to the 12th month?
 - What is the average change of this rate during the first 5 months?

8.1 Average Value of a Function

Practice

Calculus

Find the average value of each function on the given interval.

1. $f(x) = x^2$ on $[2, 4]$

2. $f(x) = \sin x$ on $[0, \pi]$

3. $f(x) = \sqrt{x}$ on $[0, 16]$

4. $f(x) = \frac{1}{x^2}$ on $[-4, -2]$

On the given interval, find the x -value where the function is equivalent to the average value on that interval.

5. $f(x) = 2x - 2$ on $[1, 4]$

6. $f(x) = -\frac{x^2}{2}$ on $[0, 3]$

Find the average rate of change on the given interval.

7. $f(x) = -(2x - 6)^{\frac{2}{3}}$ on $[1, 3]$

8. $y = x^3 - 2x^2 + 2$ on $[-1, 1]$

Find where the instantaneous rate of change is equivalent to the average rate of change. (MVT)

9. $y = x^2 - 4x + 3$ on $[0, 4]$

10. $y = \sqrt{9 - 8x}$ on $[-2, 0]$

11. **Calculator active problem.** The temperature (in °F) t hours after 9 AM is approximated by the function $T(t) = 50 + 14 \sin \frac{\pi t}{12}$. Find the average temperature during the time period 9 AM to 9 PM.

12. **Calculator active problem.** The depth of water in Mr. Brust's hot tub can be represented by the formula $h(t) = 2 - \cos(t)$, where t is the time in minutes since he begins pouring in water and $h(t)$ is measured in feet. What is the average depth of the water during the first three minutes? Set up the expression and use a calculator to help solve.

8.1 Average Value of a Function

13. **Calculator active problem.** The temperature outside during a 12-hour period is given by

$$T(h) = 60 - 5 \cos\left(\frac{\pi h}{8}\right), \quad 0 \leq h \leq 12$$

Where $T(h)$ is measured in degrees Fahrenheit and h is measured in hours. Find the average temperature, to the nearest degree Fahrenheit, between $h = 2$ and $h = 9$.

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14. Find the number(s) b such that the average value of $y = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal 3. *Hint:* quadratic formula needed!

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15. **Calculator active problem.** Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 37 - 6 \cos\left(\frac{t}{3}\right) \text{ for } 0 \leq t \leq 20,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
- What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.